ThroatUnwrap: Optimal ALTF Geometry

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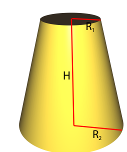
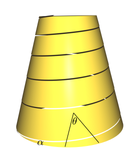
1 Method Description



The goal of this method is to create a 2D patch that could be folded into a cylindrical shape depicted in the inset without any stretch. We are given the bottom radius R_1, top radius R_2, and height H, and we also assume that radius changes linearly with the height. To solve this problem we propose an algorithm for unfolding the depicted surface into a flat patch that is isometric to the original 3D shape.



The first challenge in this unfolding procedure is to cut the surface to turn it into a topological disk. As long as cut leads to desired topology, the resulting surface will be developable, so there is no unique or optimal solution to this problem. We chose to provide the user of our system with a single parameter α that defines the angle at which the surface is cut relatively to the flat plane, as depicted in the inset. We describe this spiral cut as a parametric function of the angle θ, where the angle can go beyond 2π spiraling upward with each loop. In particular we describe the position on the cut as:

$$\boldsymbol{x}(\theta) =< r(\theta)cos(\theta), r(\theta)sin(\theta), h(\theta) > \tag{1}$$

Note that we can describe radius as a function of height:

$$r(\theta) = R_1 + \frac{R2 - R1}{H} h(\theta) \tag{2}$$

In order to find $h(\theta)$ we first observe that in order to have the desired cut angle α, we need to have $\frac{dh}{ds} = tan(\alpha)$, where s is arclength of the spiral, and $\frac{ds}{d\theta} = r$. By chain rule, we also have:

$$\frac{dh}{d\theta} = \frac{dh}{ds} \cdot \frac{ds}{d\theta} = tan(\alpha) \cdot r(h) \tag{3}$$

Which yields the first-order ODE:

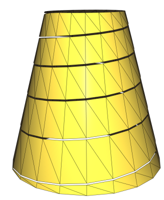
$$\frac{dh}{d\theta} + c_1 h = c_2 R_1 \tag{4}$$

$$
\begin{aligned}
c_1 &= -tan(\alpha)\frac{R_2 - R_1}{H} &(5)\\
c_2 &= tan(\alpha)R_1 &(6)
\end{aligned}
$$

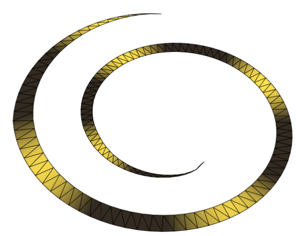
Solving it using integrating factor gives:

$$h(\theta) = \frac{c_2}{c_1} - \frac{c_2}{c_1}e^{-c_1\theta} \tag{7}$$

We now define the unfolding algorithm. First, we triangulate the surface described above (along with the cut). We can now traverse the triangles along the increasing θ iteratively unfodling each triangle. Since each triangle is connected to a previous one only at one edge we trivially find the unique rotation that aligns each next triangle in the same plane as the preceding triangle and does not cause the triangle to fold over the preceding one.



Note that there is only one such solution. This yields the final 2D panel shown in the inset. Unwarping this spiraling patch into something more slliptical will cause some stretch in the triangles.



2 Results

We now demonstrate some cut angles and resulting flattenings in Figure 1.

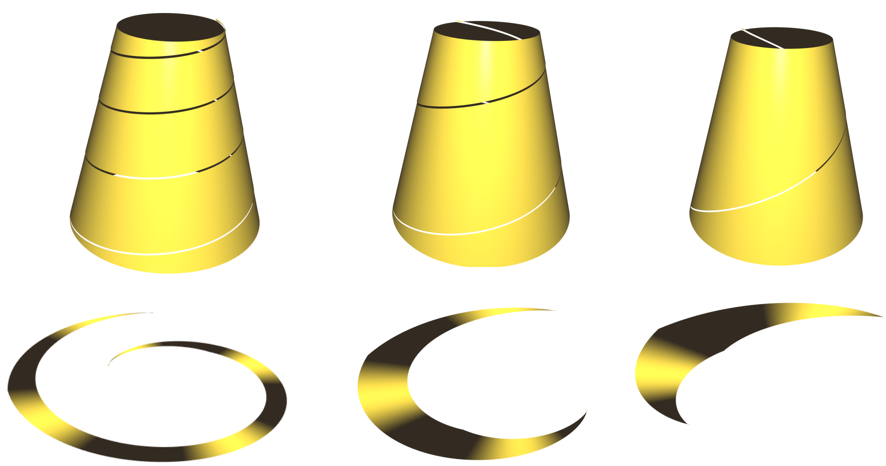


Figure 1: Here are some examples of cut angles (see top spirals) and resulting flattenings (see bottom).