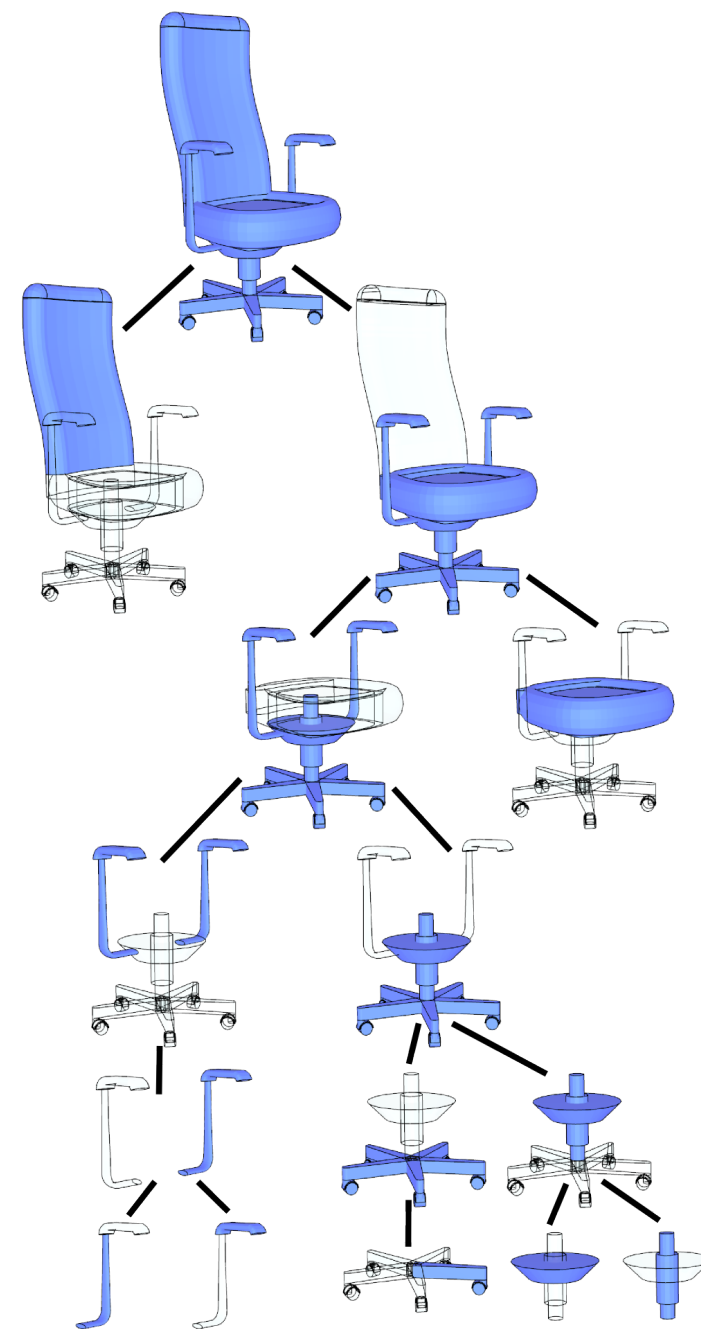
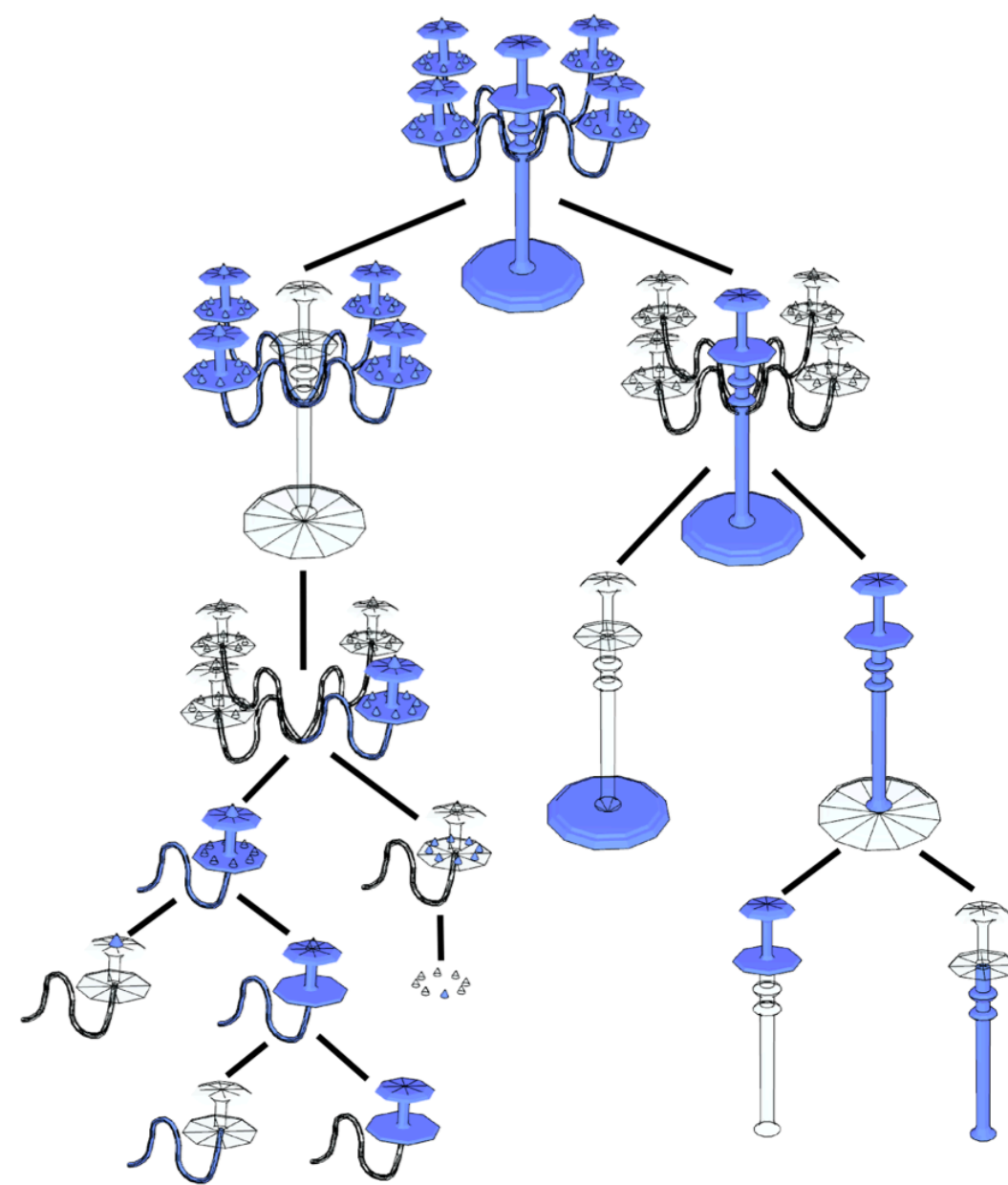


- **Introduction to Geometric ‘Structure’**
- **Extracting Structures**
  - analysis of Individual Models
  - analysis of Shape Collections (co-analysis)
  - Structural Hierarchies
- **Manipulating Structures**
  - Modeling as Structural Variations
  - Structure-guided Design
  - Organization + Exploration of Shape Collections
- **Future Directions**

# Structural Hierarchies



*Niloy J. Mitra*

*Michael Wand*

*Hao Zhang*

*Daniel Cohen-Or*

*Vladimir Kim*

*Qi-Xing Huang*



Universiteit Utrecht

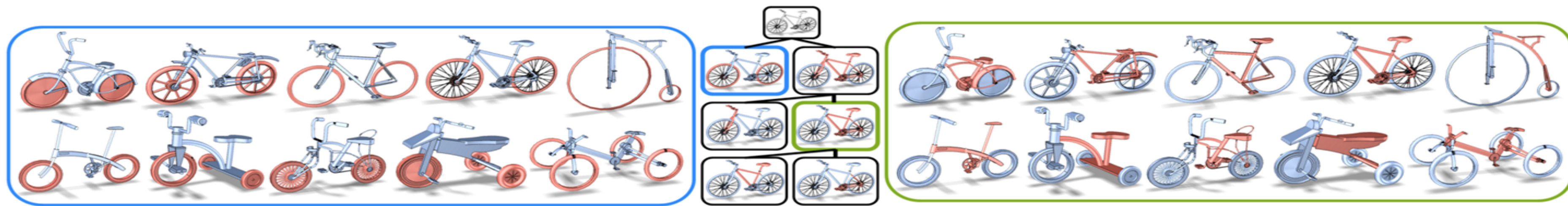


TEL AVIV UNIVERSITY



# Why hierarchy?

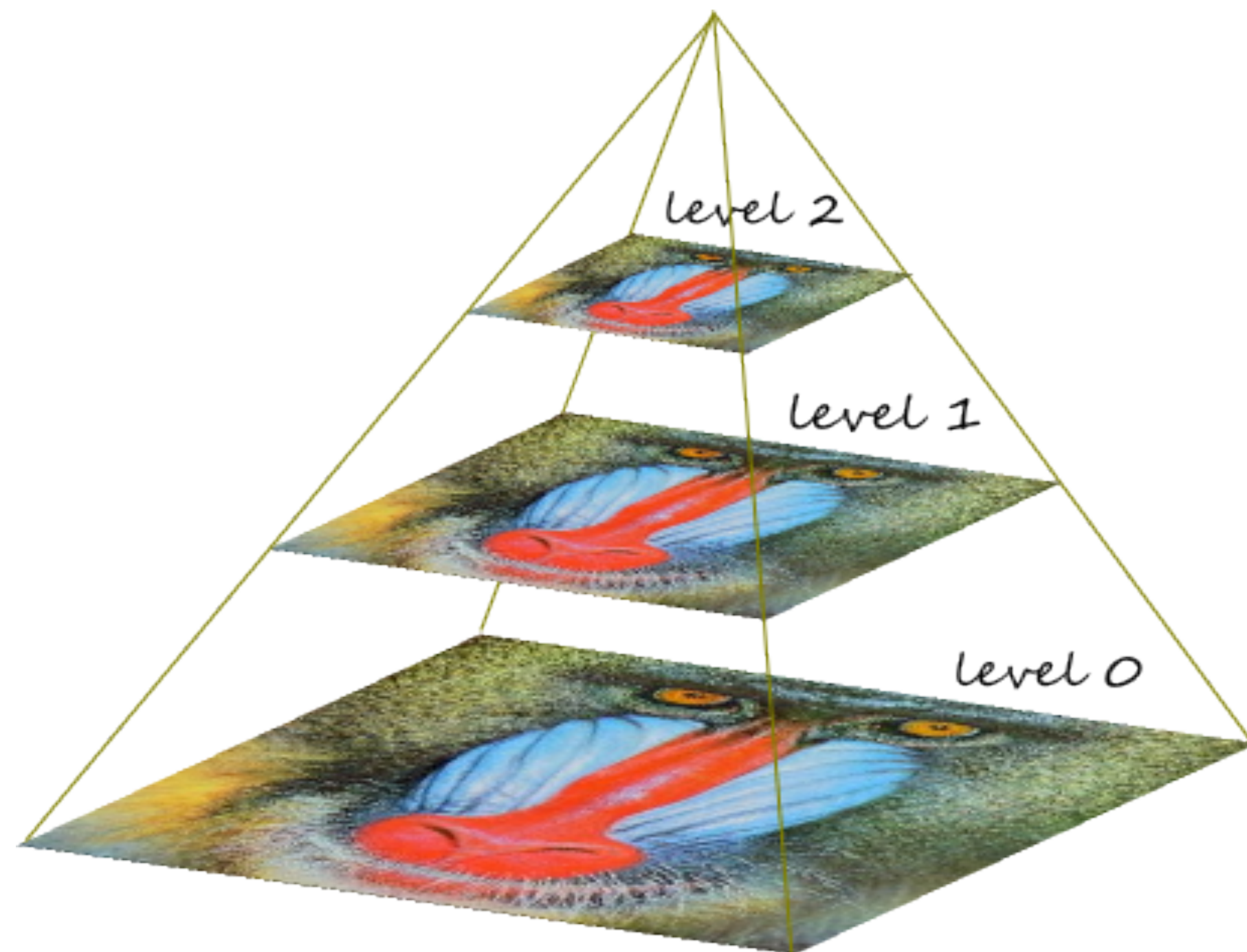
- **Efficiency and flexibility:** hierarchy offers BOTH high-level abstraction and granularity
- **Natural choice:** human **perception** of structures IS hierarchical [Palmer 1977, Hoffman & Richards 1984]
- Appropriate model to capture **diversities in a set**: coarse levels encode commonalities; finer details lower in the tree



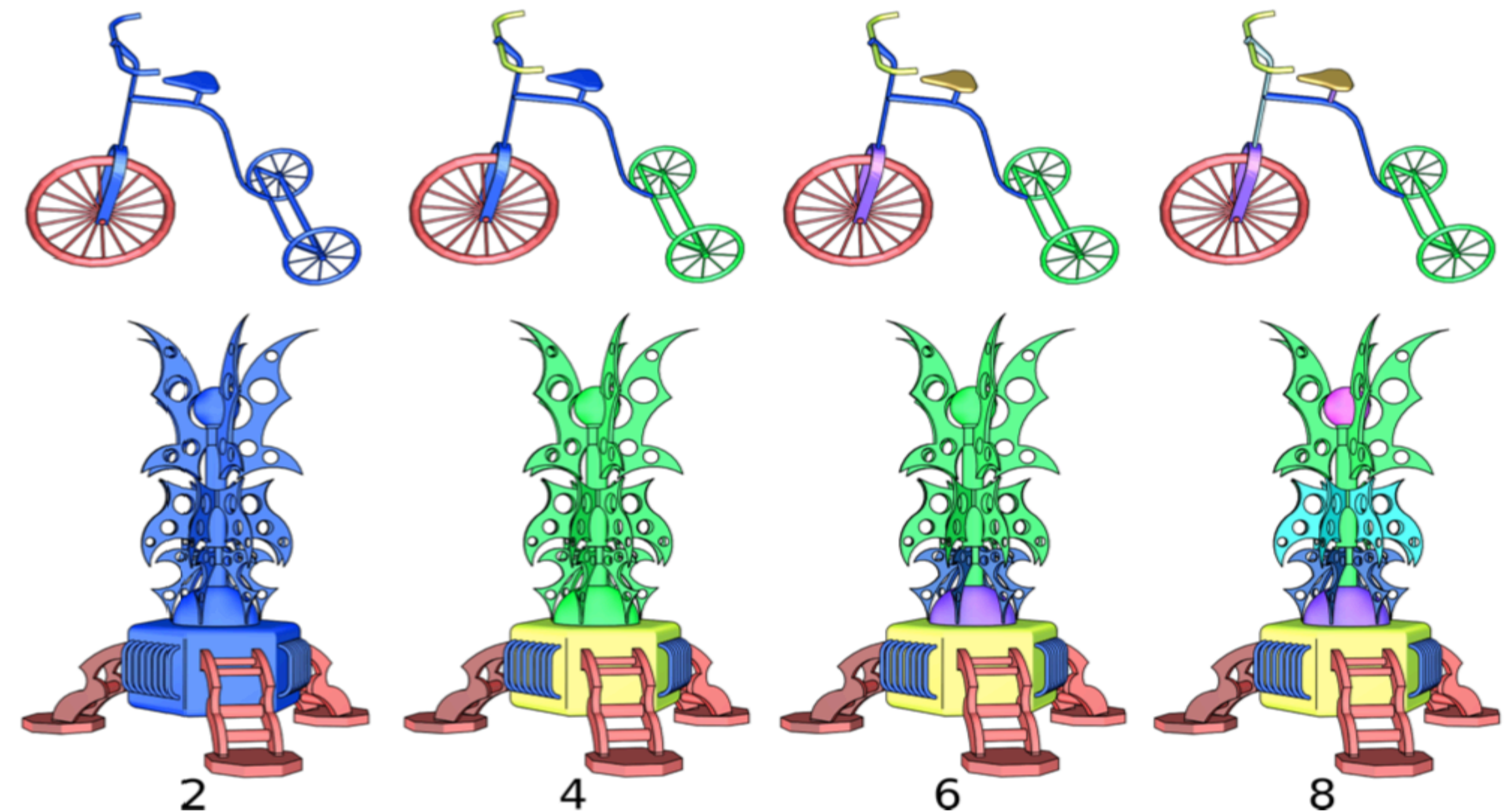


# Familiar examples

- Image pyramids



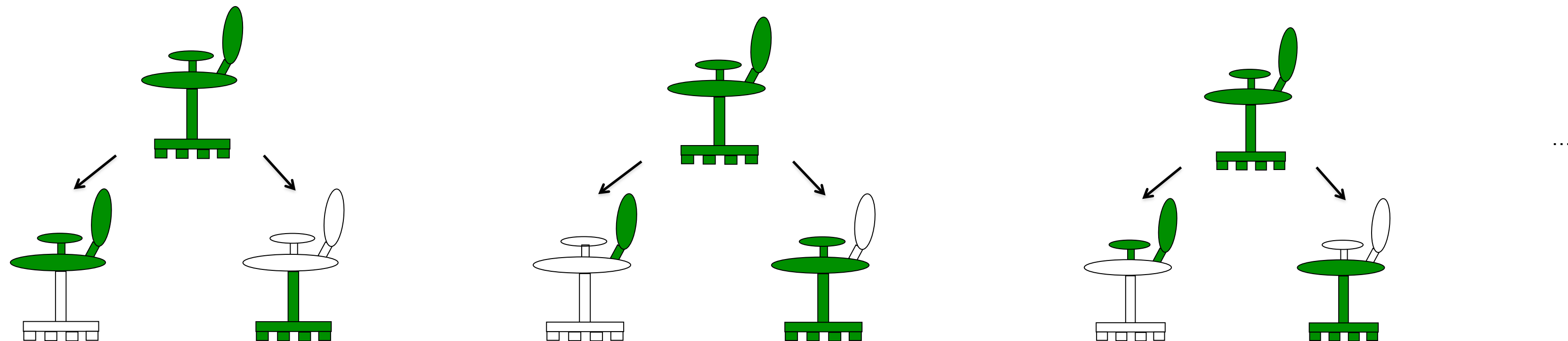
- Hierarchical segmentation





# Key question

- What is **the right structural hierarchy**?
- **Challenges:**
  - Many possible hierarchies for a given shape: larger search space than (flat) shape segmentation
  - Many possible criteria for “best” structural hierarchy
  - Even human observers may disagree on what is the ground truth



# We are interested in ...

- ... not just any structural hierarchy
- ... but one that **best explains** one or a set of structures
- Exploit power of **symmetry** for analyzing individual shapes [Martinet 2007, Simari 2006, Wang 2011, Zhang 2013]
- Exploit power of a set for **co-hierarchical analysis** [van Kaick 2013]



# Hierarchy construction

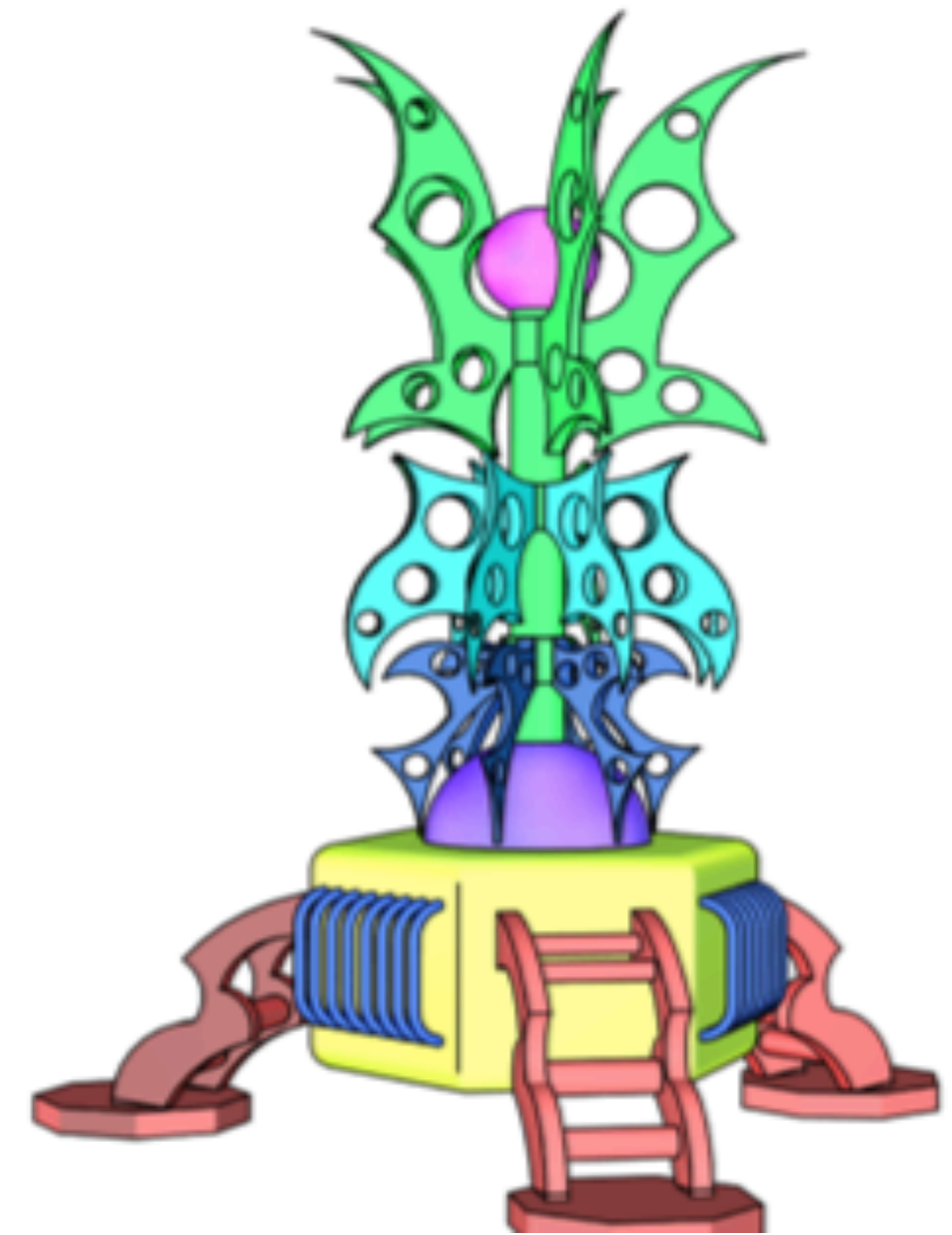
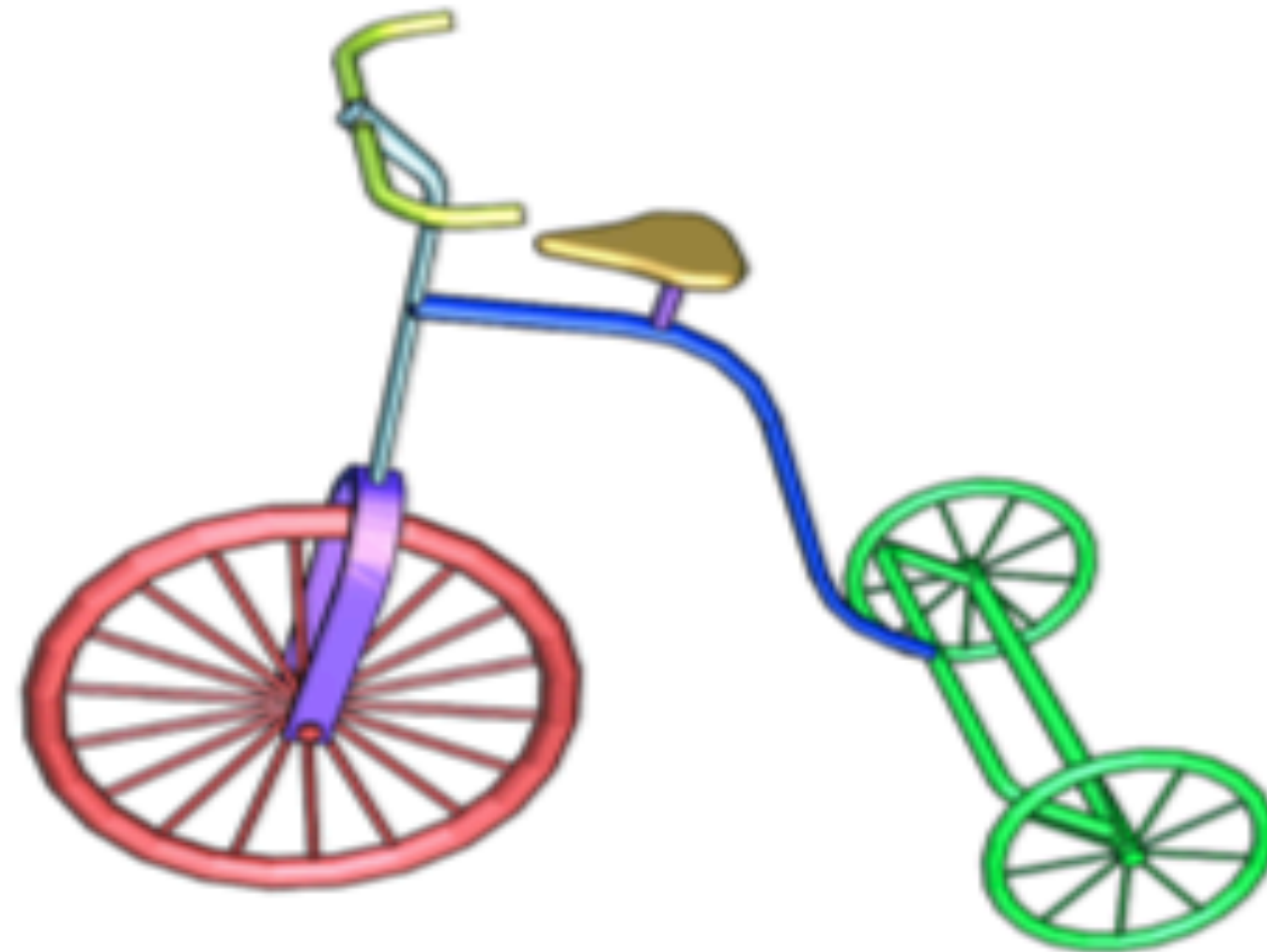
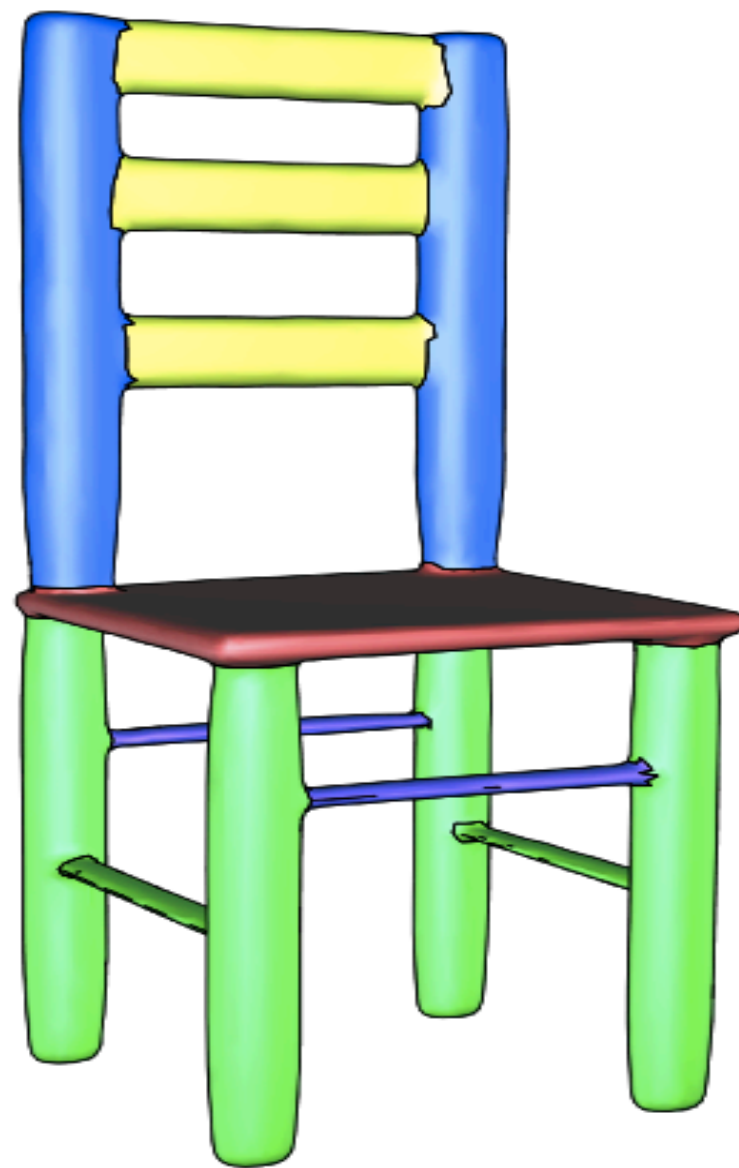
Two basic solution paradigms:

- **Top-down**
  - Recursively split or divide an input shape
- **Bottom-up**
  - Start with a fine segmentation into primitive parts
  - Then recursively group parts, e.g., via graph contraction

Key ingredient: **how to prioritize the operations**

# Why symmetry?

- Symmetric parts tend to perform **the same function**



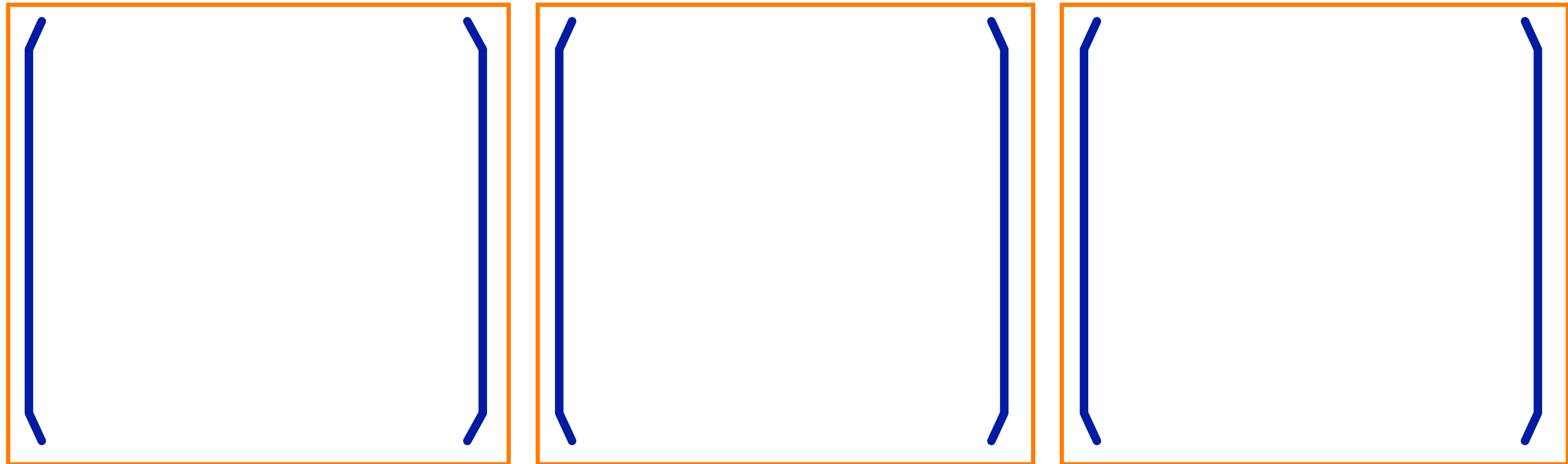


# Why symmetry?

- Symmetry leads to “better explanation” of structures
  - With symmetry comes redundancy
  - Removing redundancy leads to more compact representations
  - A compact representation tends to be the right representation
  - Occam’s Razor: other things being equal, the simplest, which is likely the most compact, explanation tends to be the best explanation
  - Works on minimum description length (MDL) follows this line of thought

# Why symmetry?

- **Perceptual grouping** via Gestalt law of symmetry



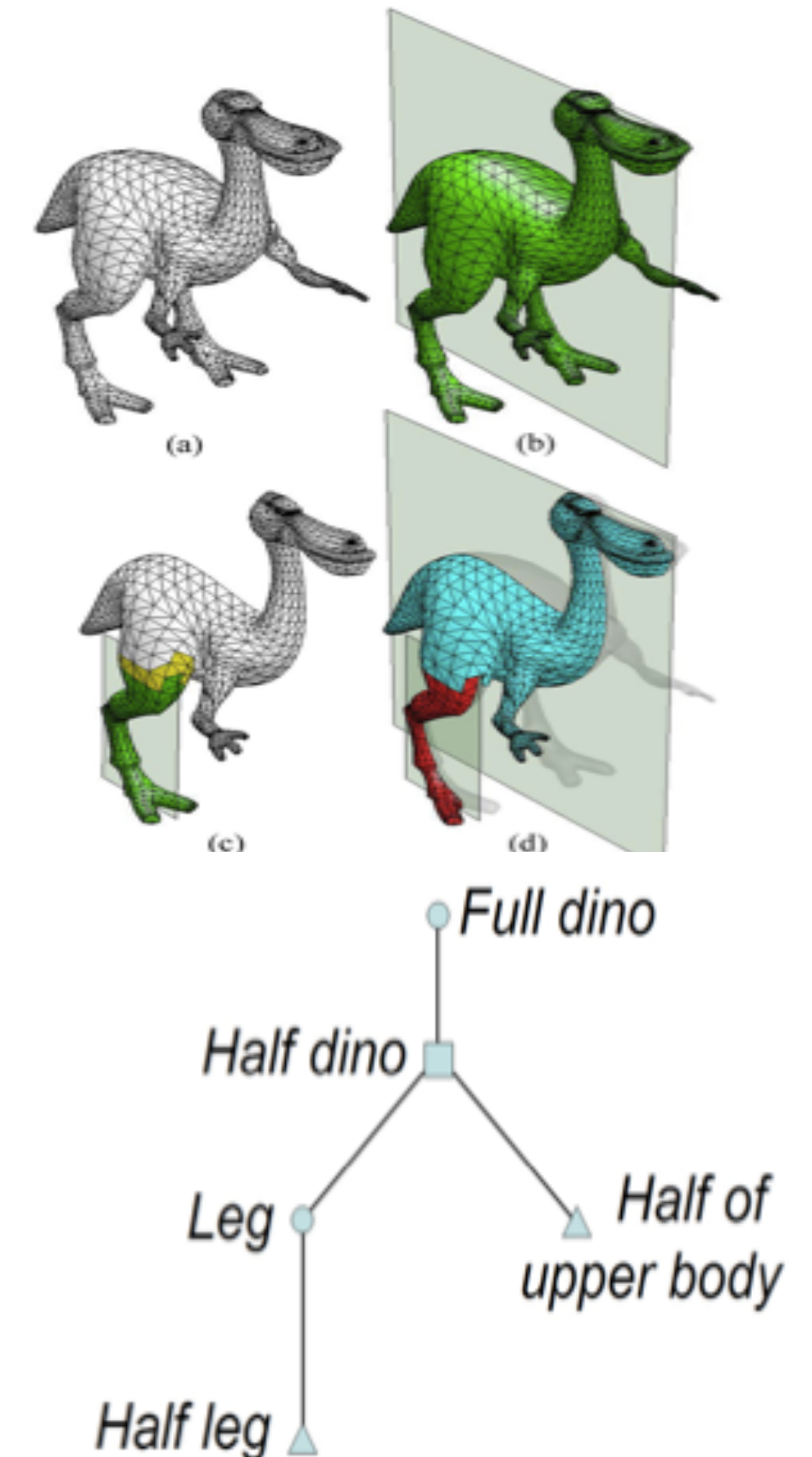


- Symmetry enables shape analysis at a **semantic** level:
  - Role in functional grouping
  - Role in offering the simplest (best) explanation
  - Role in perceptual grouping
- However, symmetry is
  - a purely **geometric** notion: explicit detection **w/o training data**
  - but requires more **global** analysis

# Folding meshes [Simari et al. 2006]

Key idea: “fold” reflection symmetries for compact shape representation

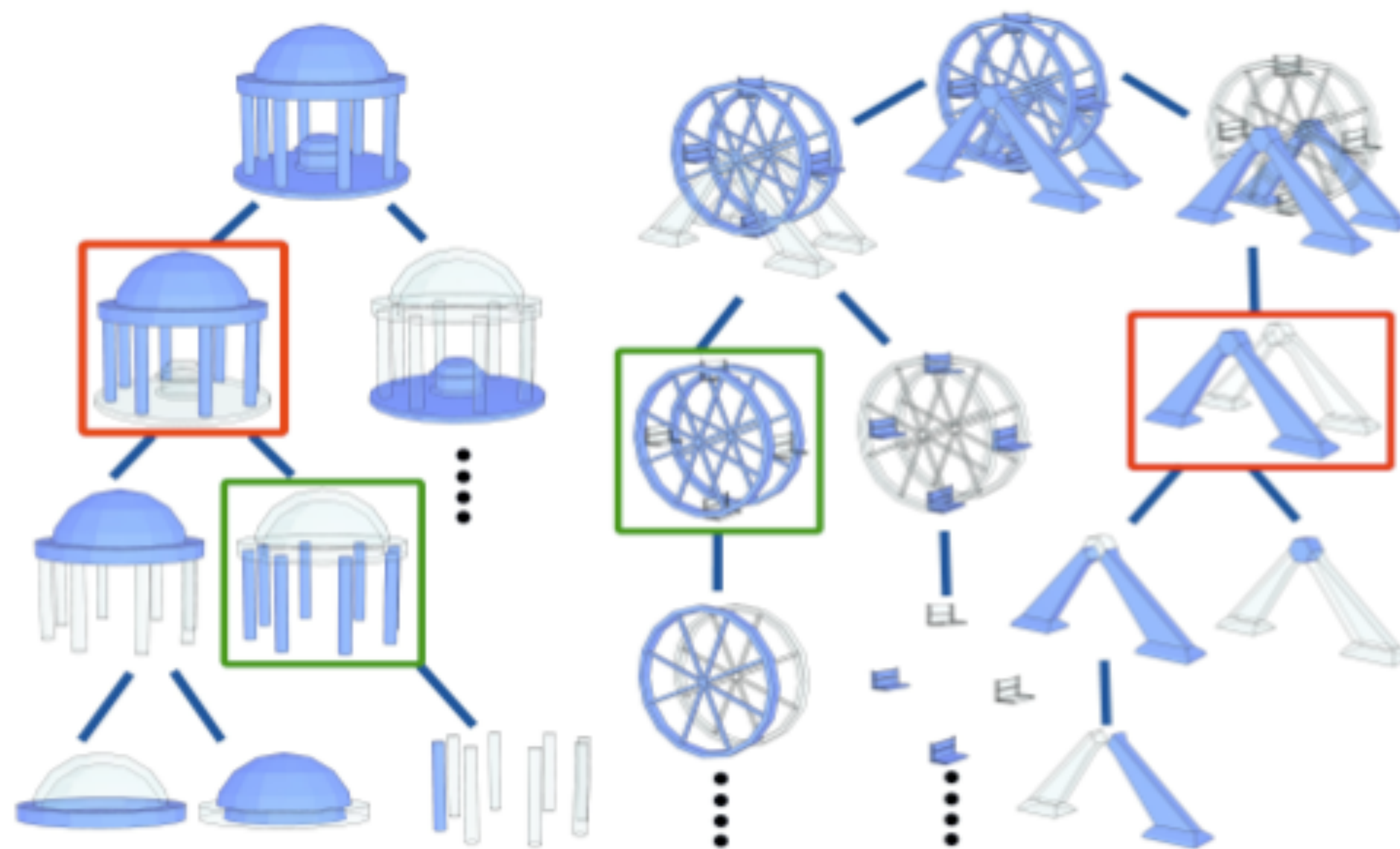
- Detect major reflection symmetry
- Divide shape into symmetric part  $A$  (full dinosaur) and the rest  $B$
- Recursively process  $B$  and **half** of  $A$
- **Top-down:** folding mesh hierarchy





# Symmetry hierarchy [Wang et al. 2011]

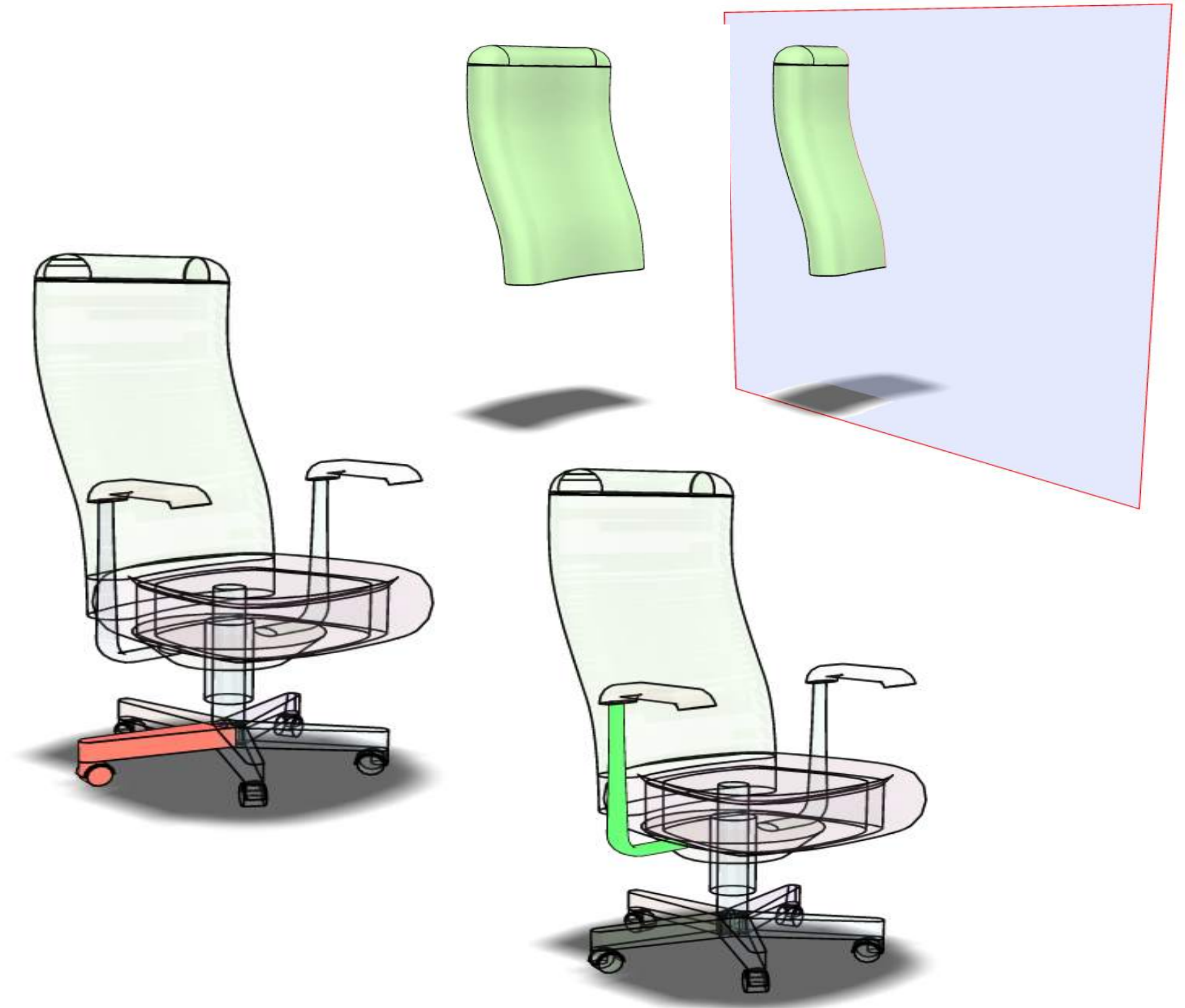
Key idea: symmetry guides **grouping and assembly** of shape parts for a meaningful hierarchical part organization.



# Hierarchy construction



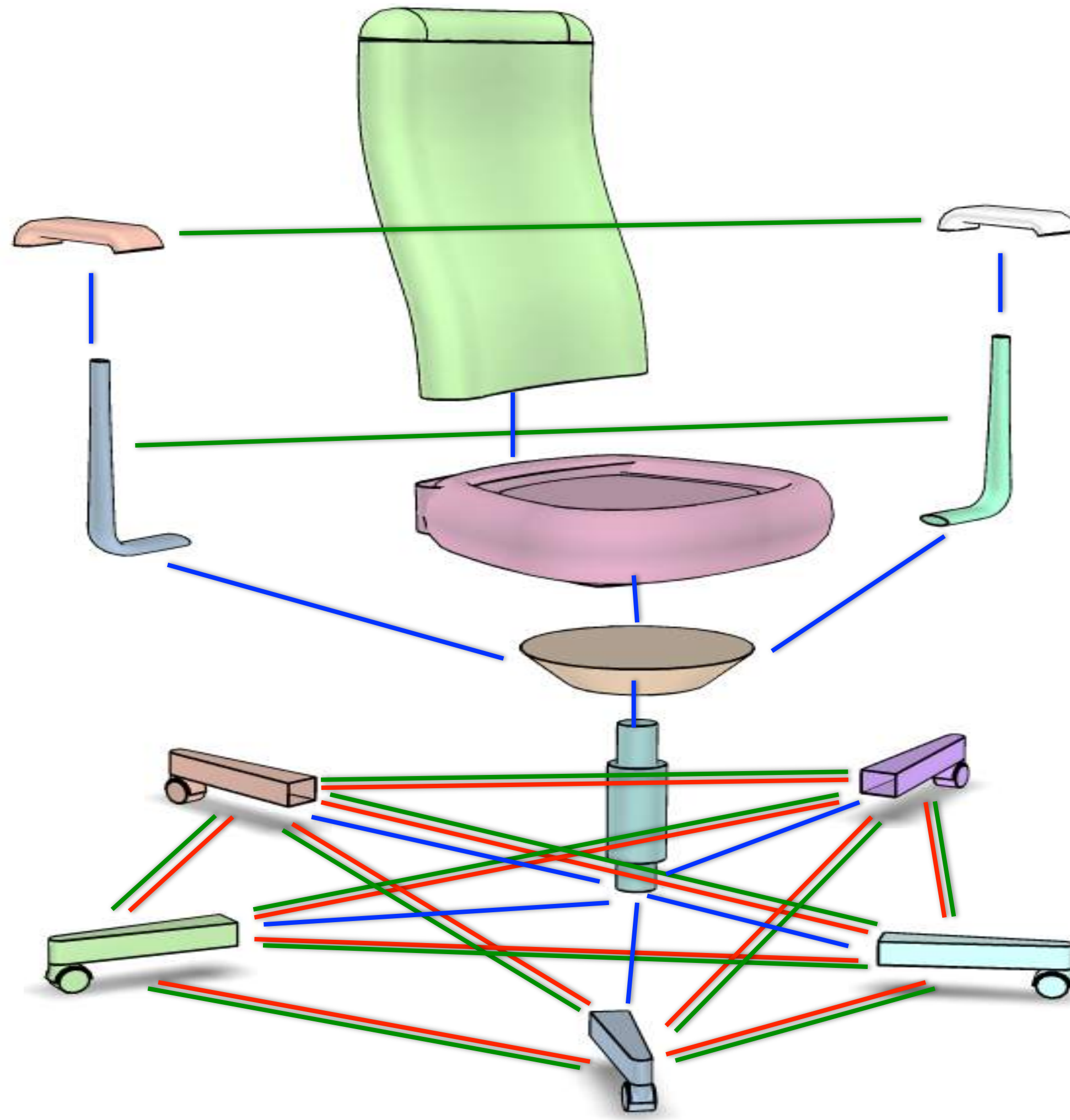
Pre-segmentation



Symmetry detection



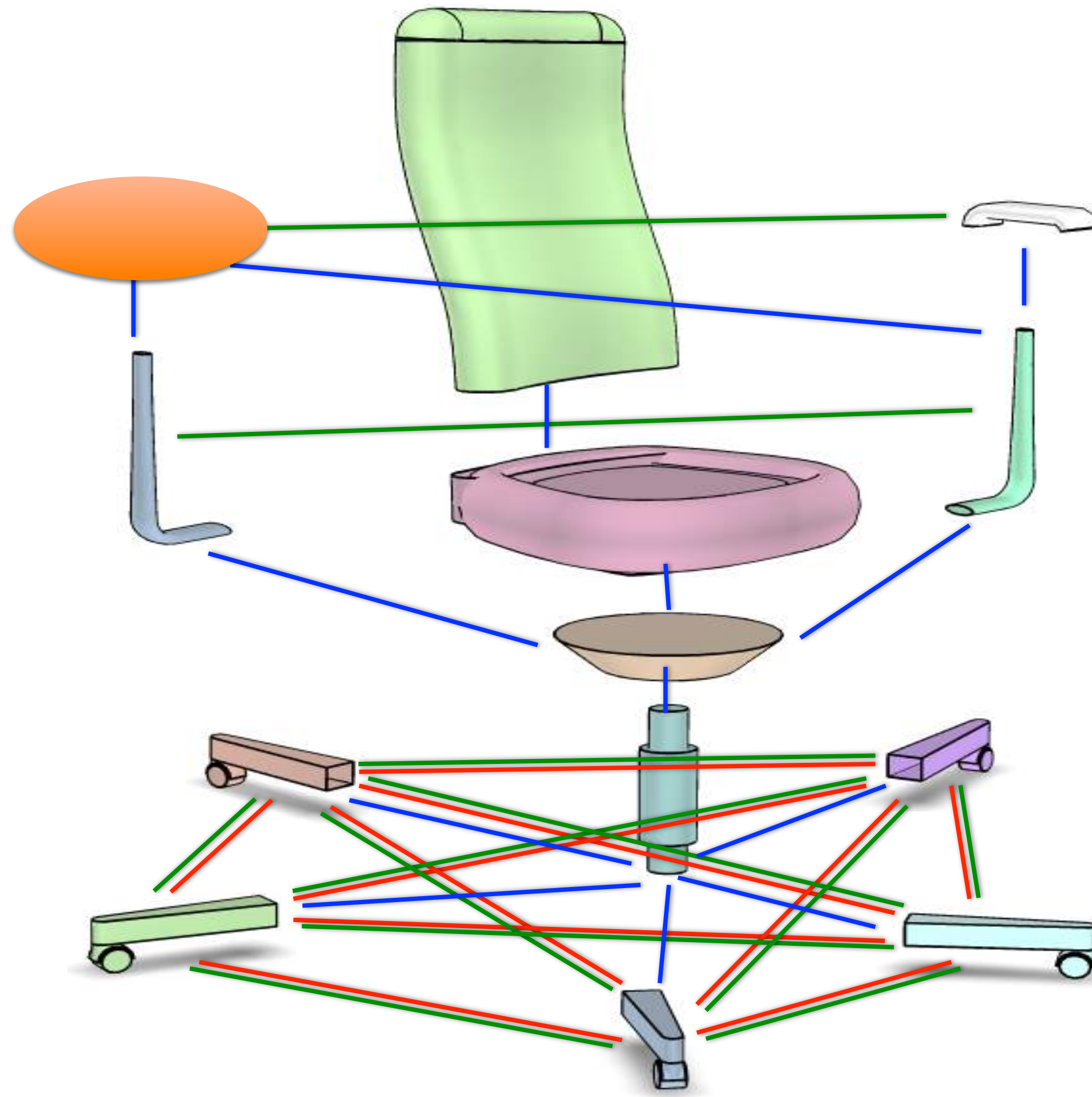
# Initial graph



- Rotational symmetry
- Reflection symmetry
- Connectivity

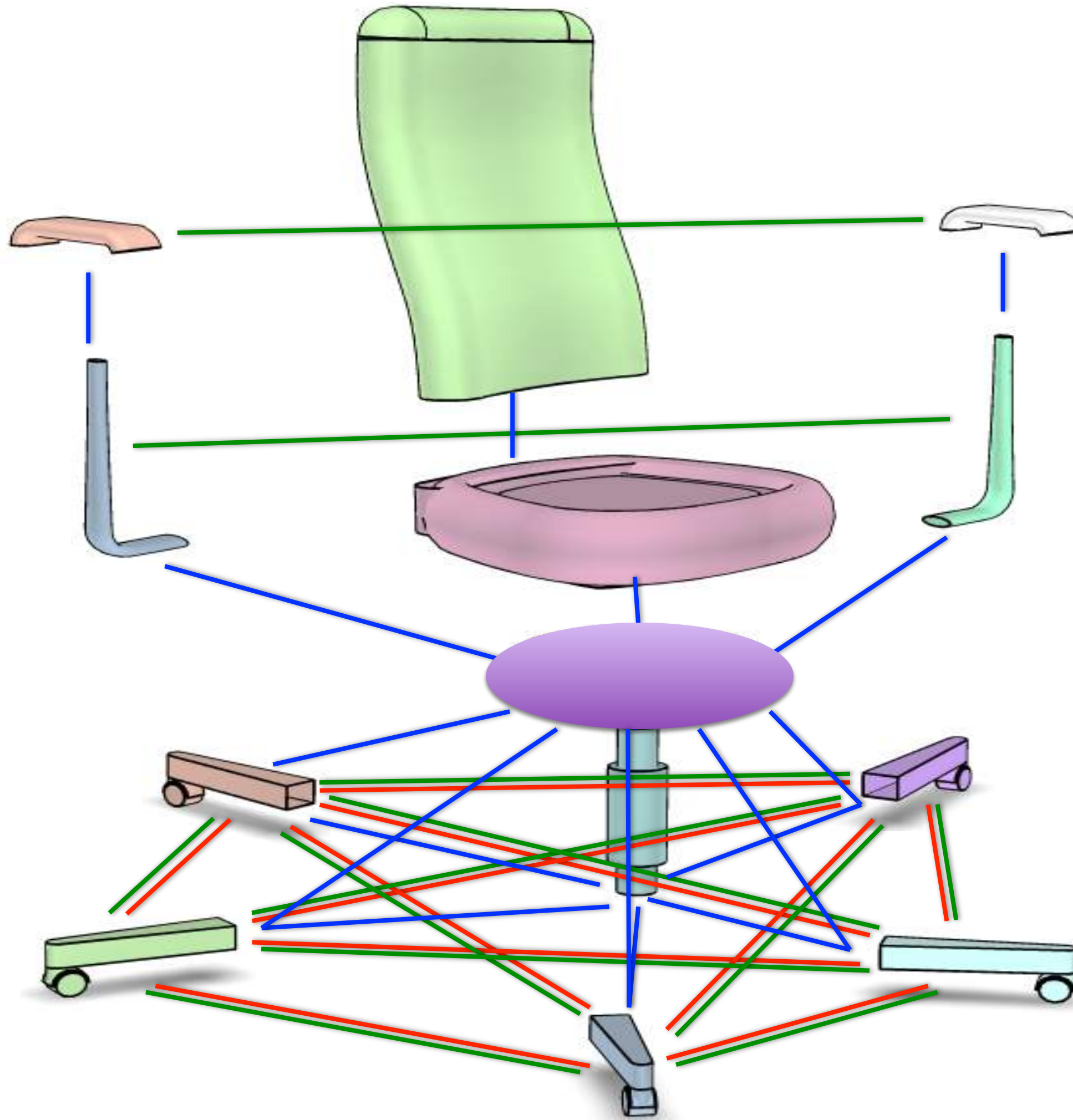


# Bottom-up graph contraction



Two operations:  
Grouping by symmetry

# Bottom-up graph contraction



Two operations:

Grouping by symmetry

Assembly by proximity

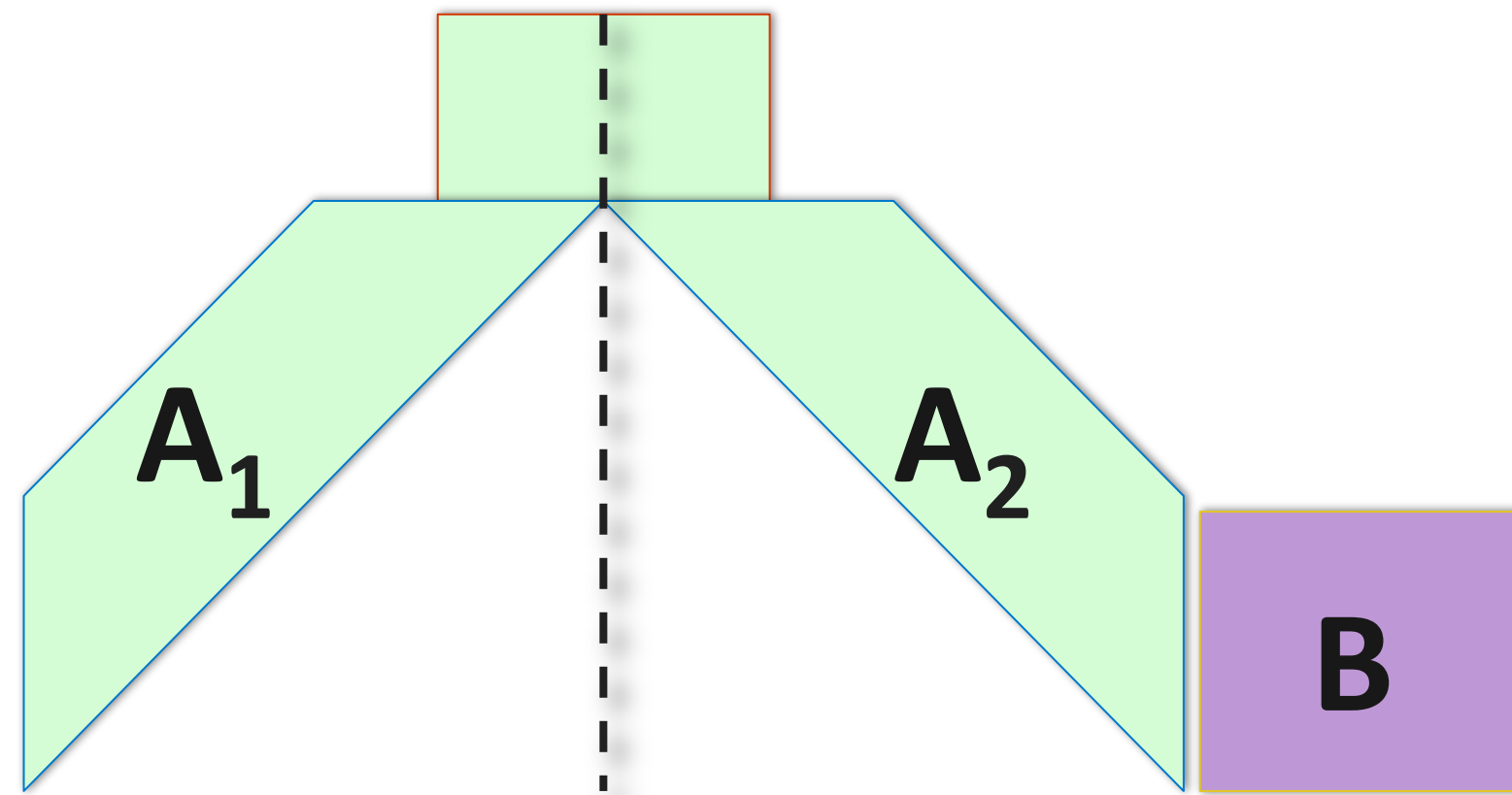
# How to order contractions?

- Guiding principles
  - Perceptual grouping: Gestalt law of symmetry
  - Compactness of representation: Occam's Razor
  - Results in a set of “precedence rules”



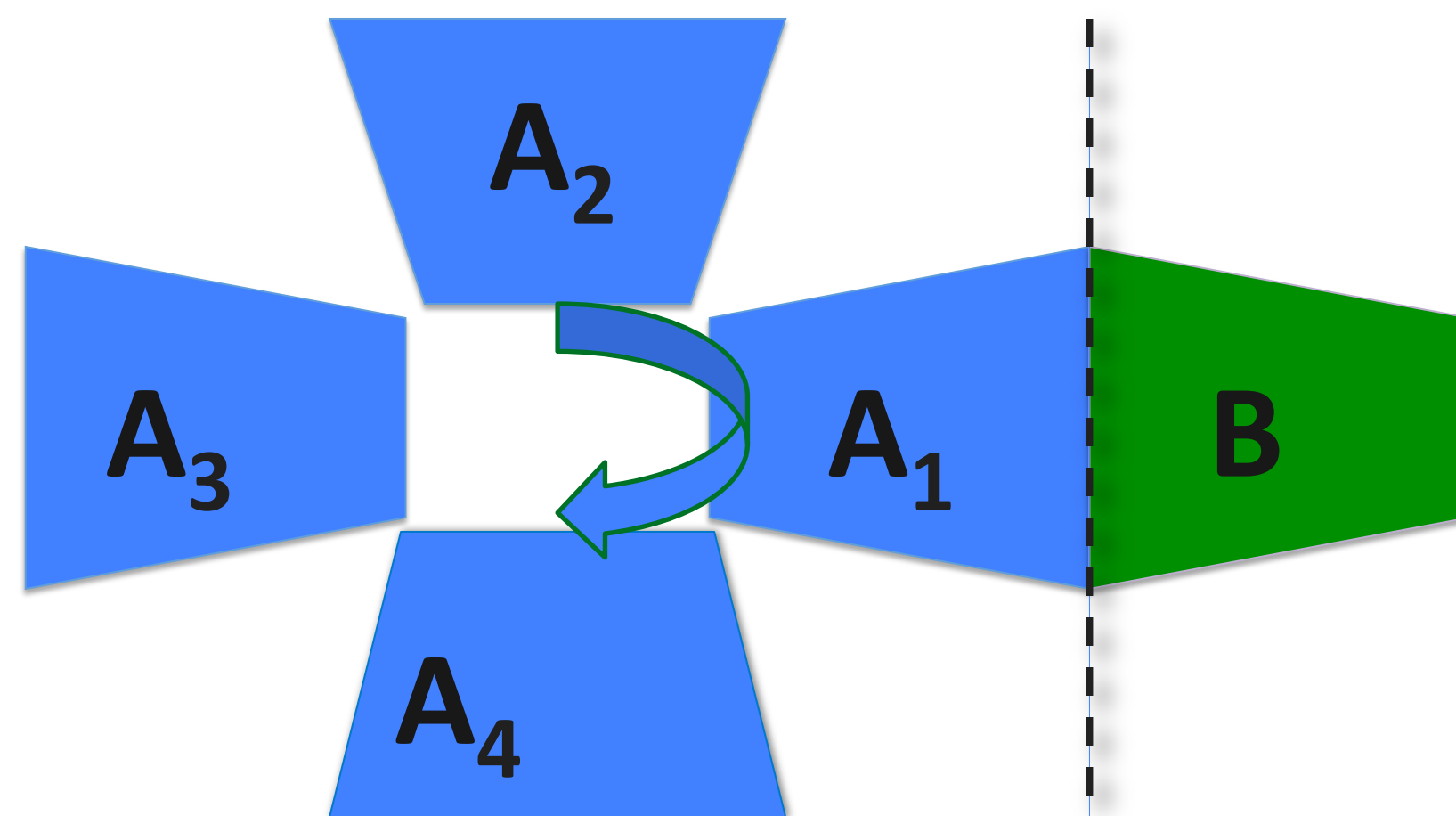
# Example of precedence rules

- Grouping-assembly mixing rules, e.g.,
  - symmetry grouping takes precedence over assembly



# Example of precedence rules

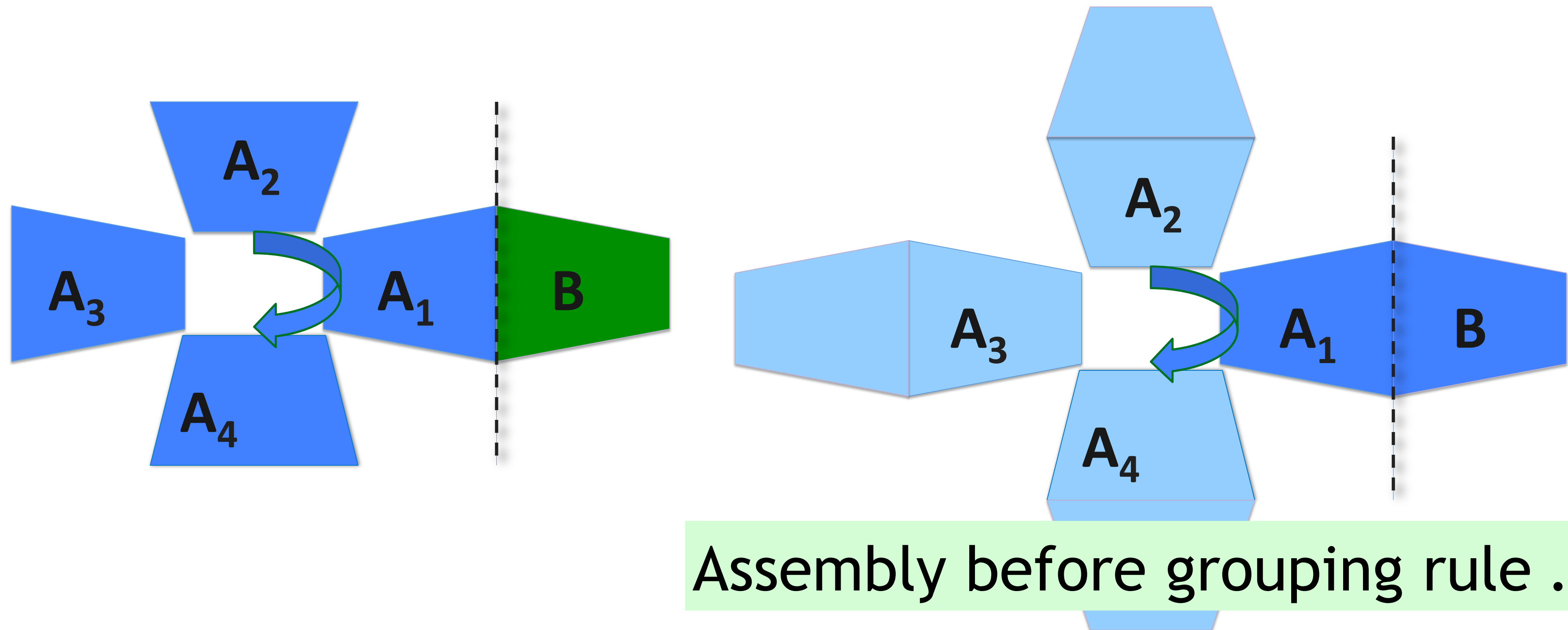
- Symmetry grouping rules, e.g.,
  - rot-symmetry takes precedence over ref-symmetry



unless

# Example of precedence rules

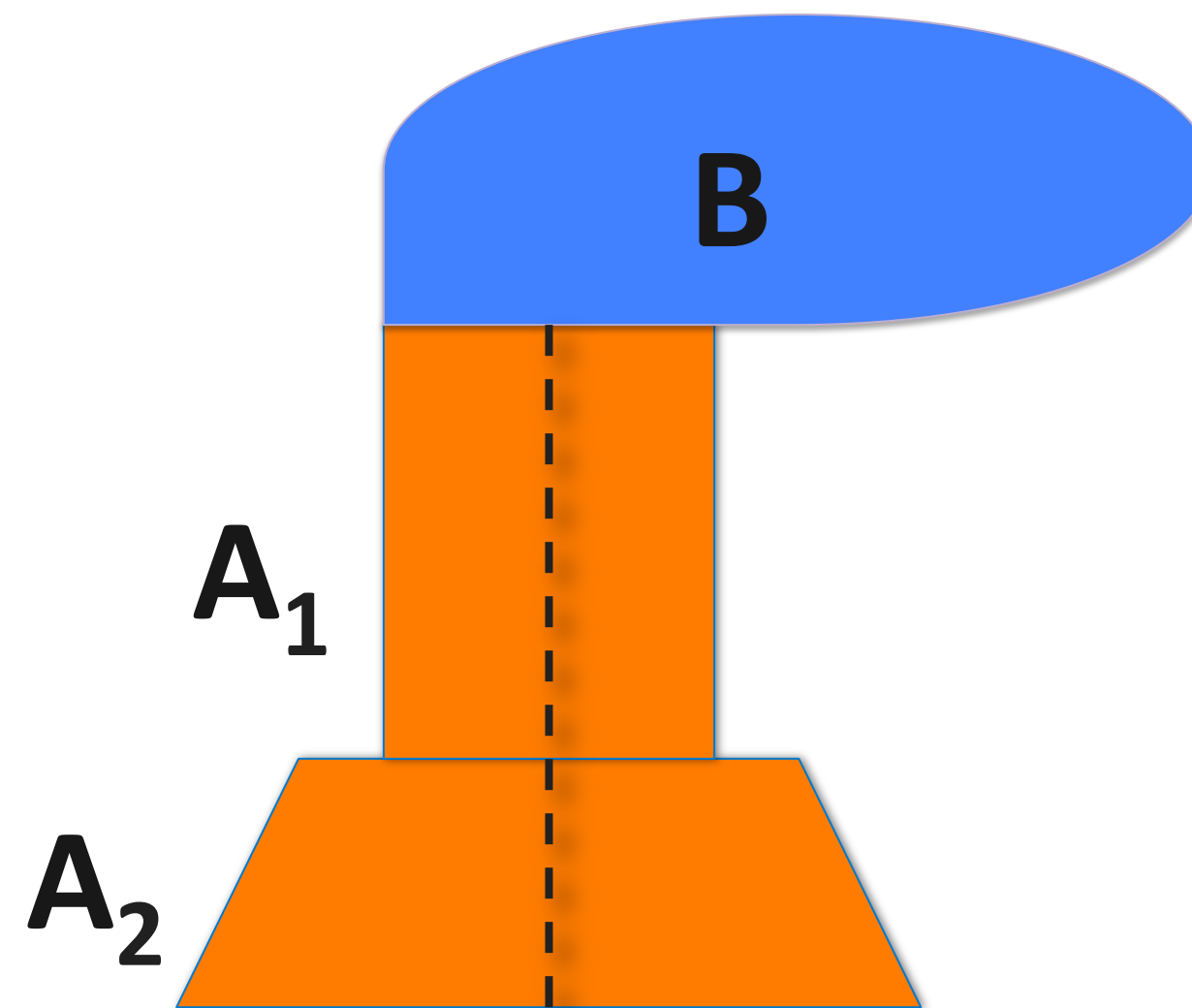
- Symmetry grouping rules, e.g.,
  - rot-symmetry takes precedence over ref-symmetry



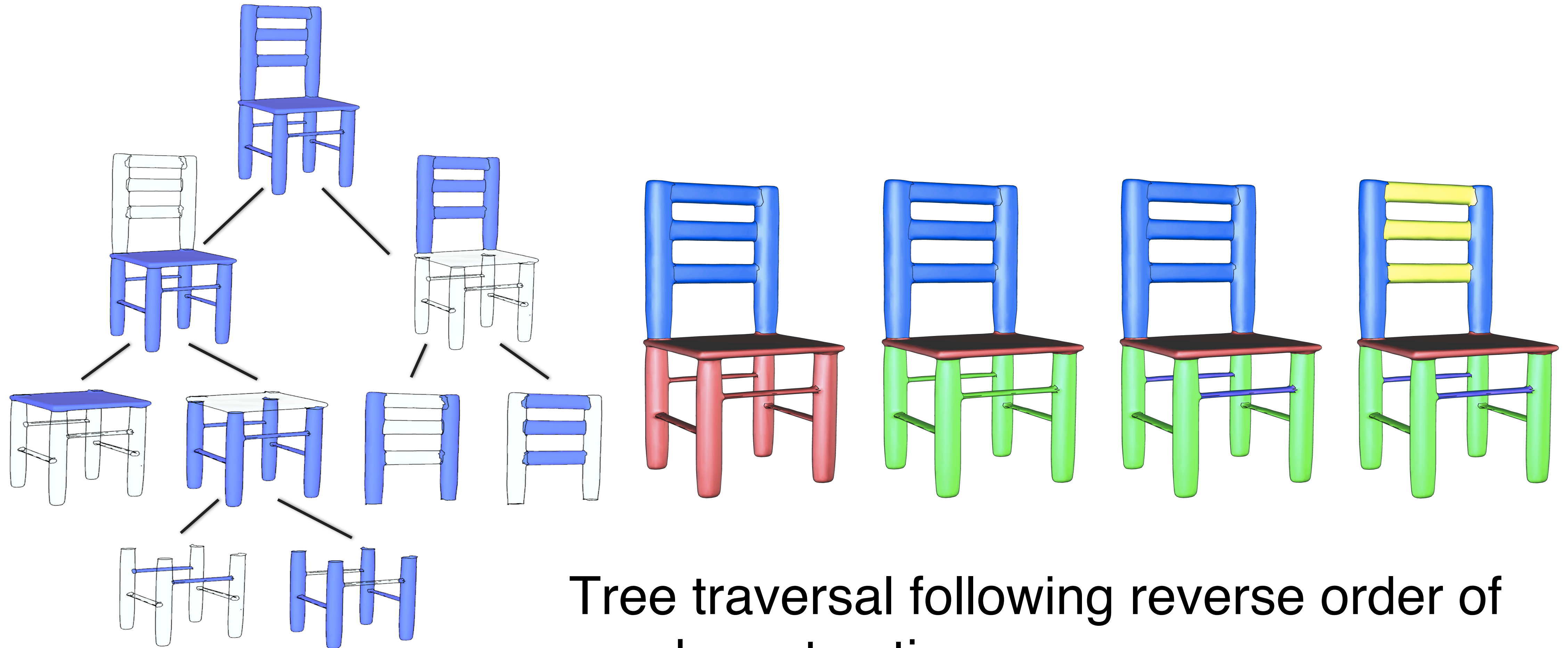


# Example of precedence rules

- Assembly rules, e.g.,
  - assemble parts that preserve more symmetries first



# App: hierarchical segmentation



Tree traversal following reverse order of graph contractions

# App: structural editing

## Structural Shape Editing



# Limitations

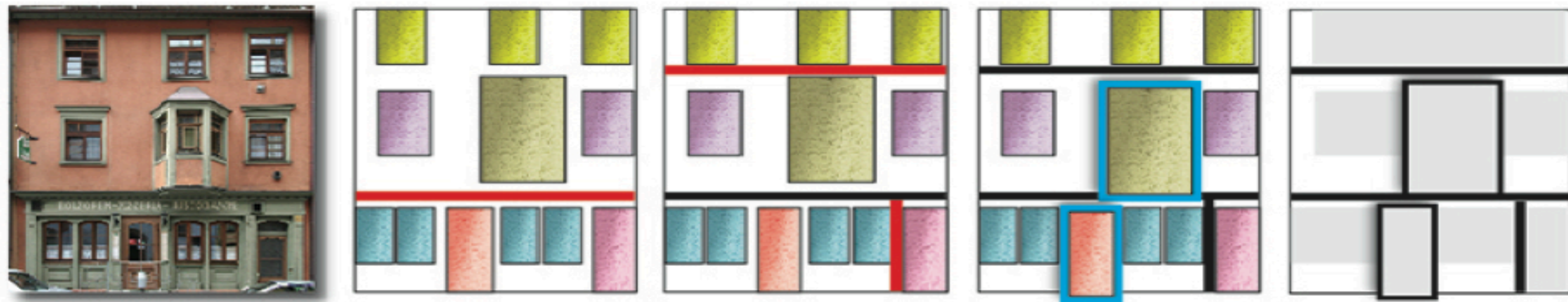
- Construction is not the result of an optimization, but based on hand-crafted precedence rules
- **Inconsistent hierarchies** for objects in same class



# Symmetry maximization [Zhang et al. 2013]

Goal: a **generative model** that best explains a structure

- Explain irregular 2D facades via **symmetry maximization**
- Use **hierarchical decomposition** as generative model

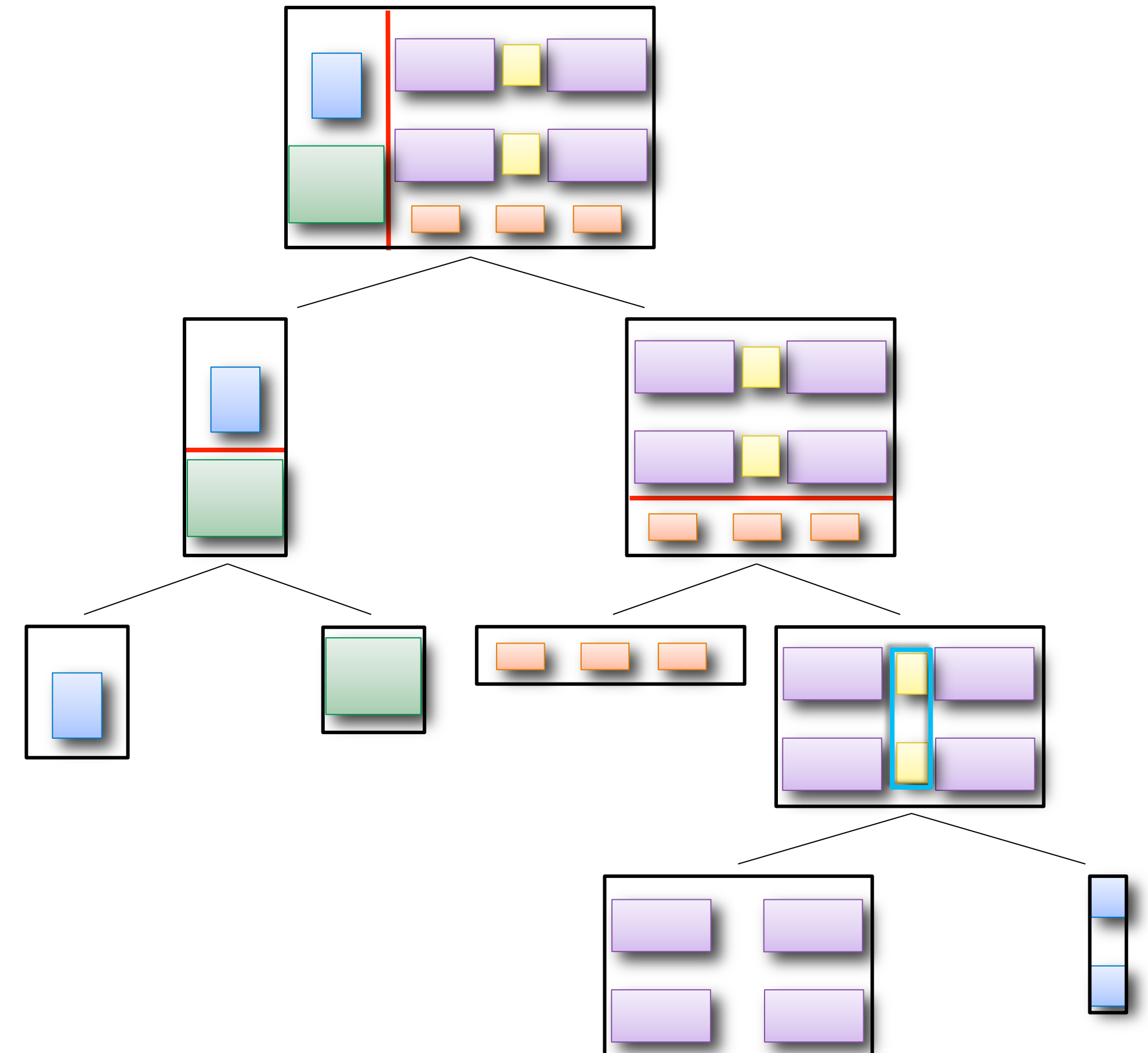
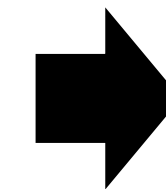


- Decomposition stops when reaching a regular grid (maximal symmetry) as it requires no explanation



# Structural hierarchy

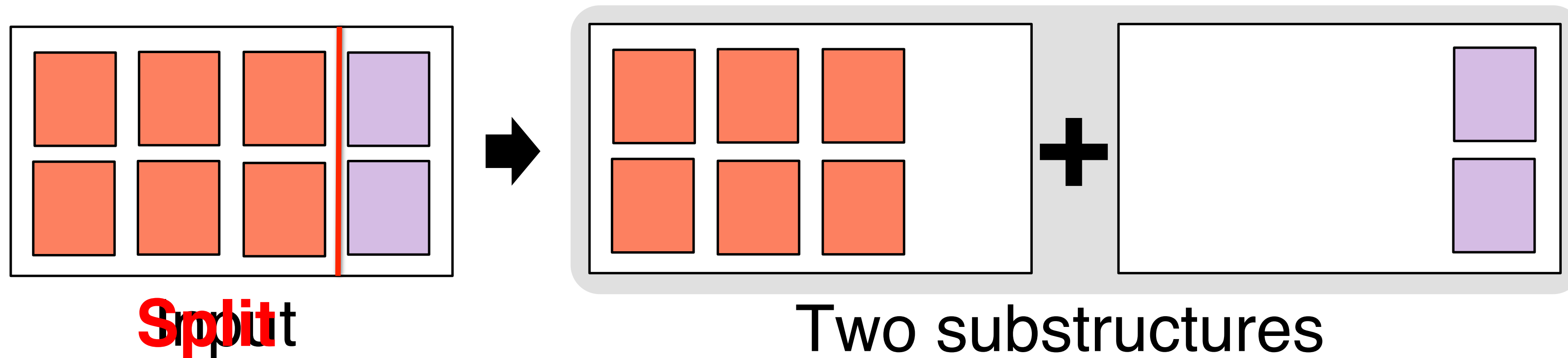
- Hierarchical 2-way decomposition





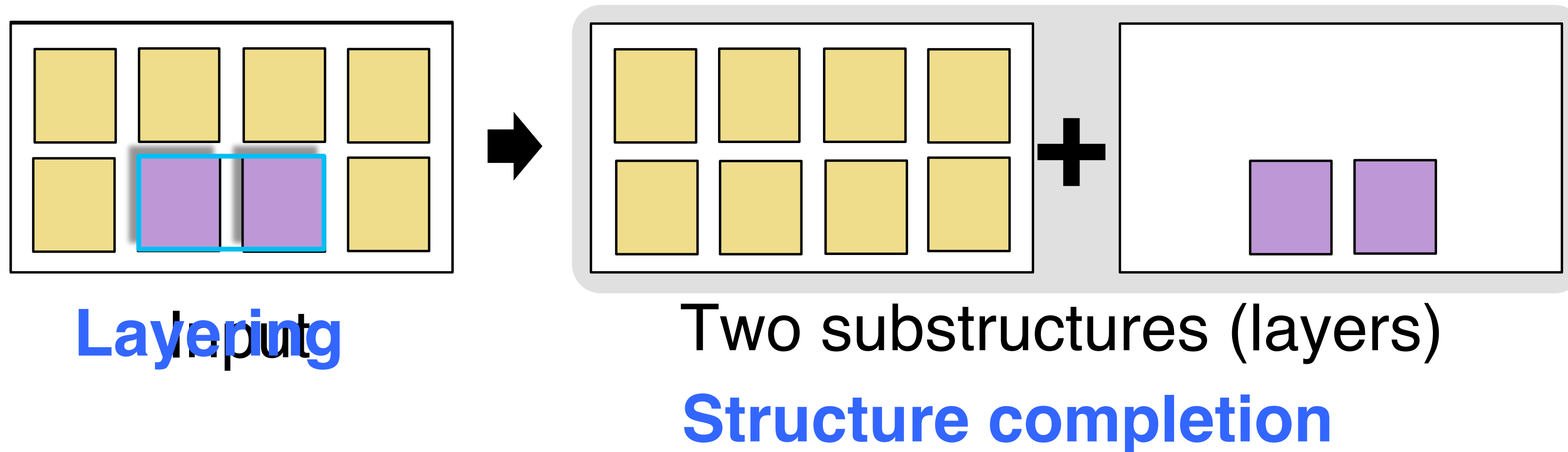
# Two operations

- 2-way splits along two lateral directions



# Two operations

- Layering: “split” along depth direction



# Layering + split vs. split only

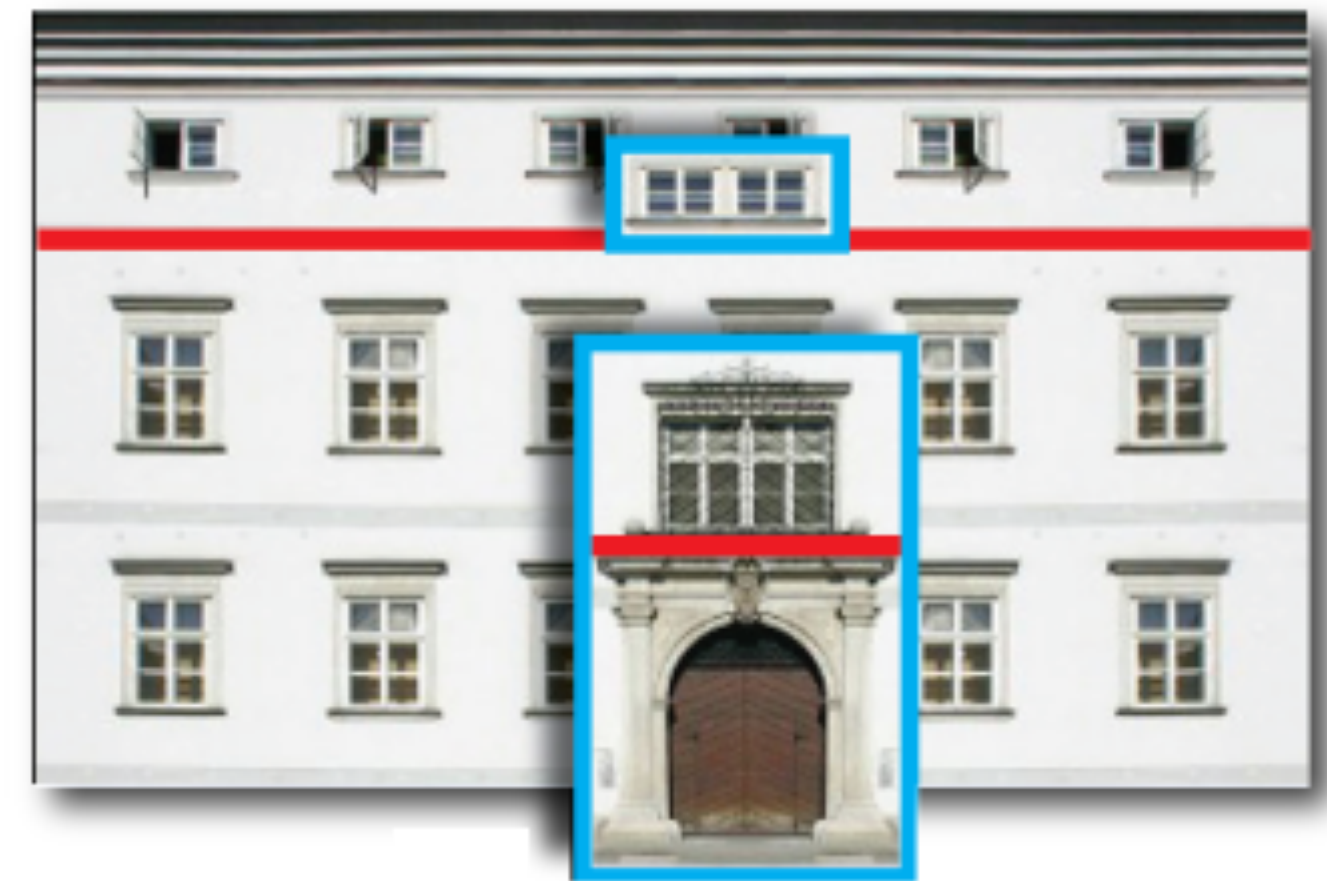
- Layering leads to more compact (simpler) explanation



Input



Split only  
8 ops.



Split + Layering  
4 ops.

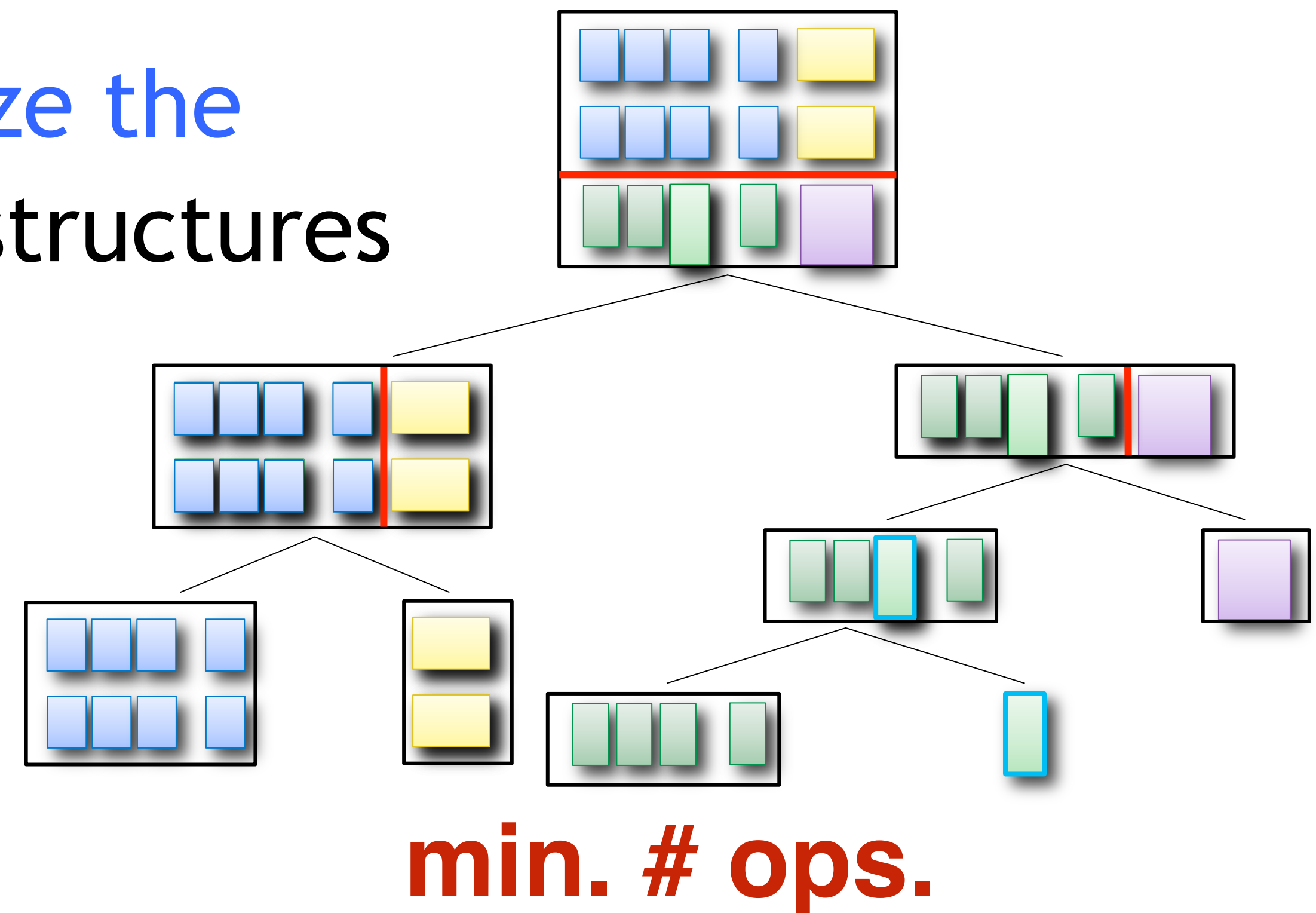


# Best explanation via SYMAX

Key ideas:

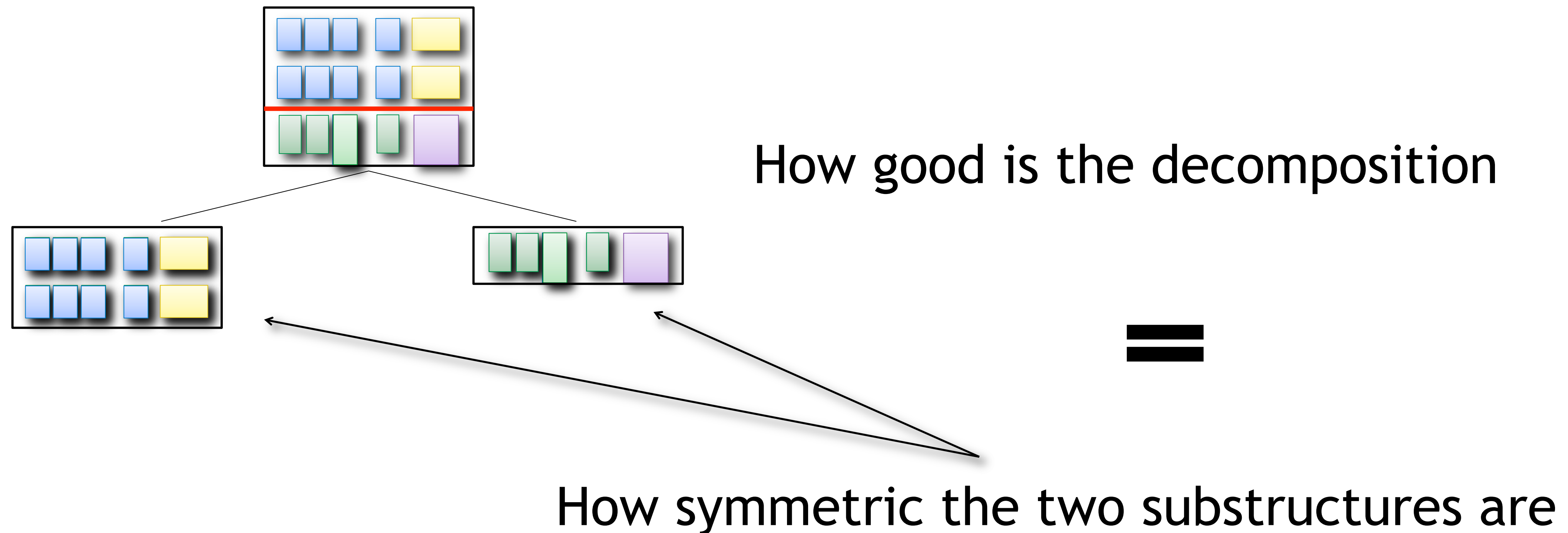
- Best explanation = simplest explanation = **fewest decompositions**
- Decomposition should then **maximize the symmetry (SYMAX)** of resulting substructures

SYMAX is consistent with  
**Gestalt law of symmetry**

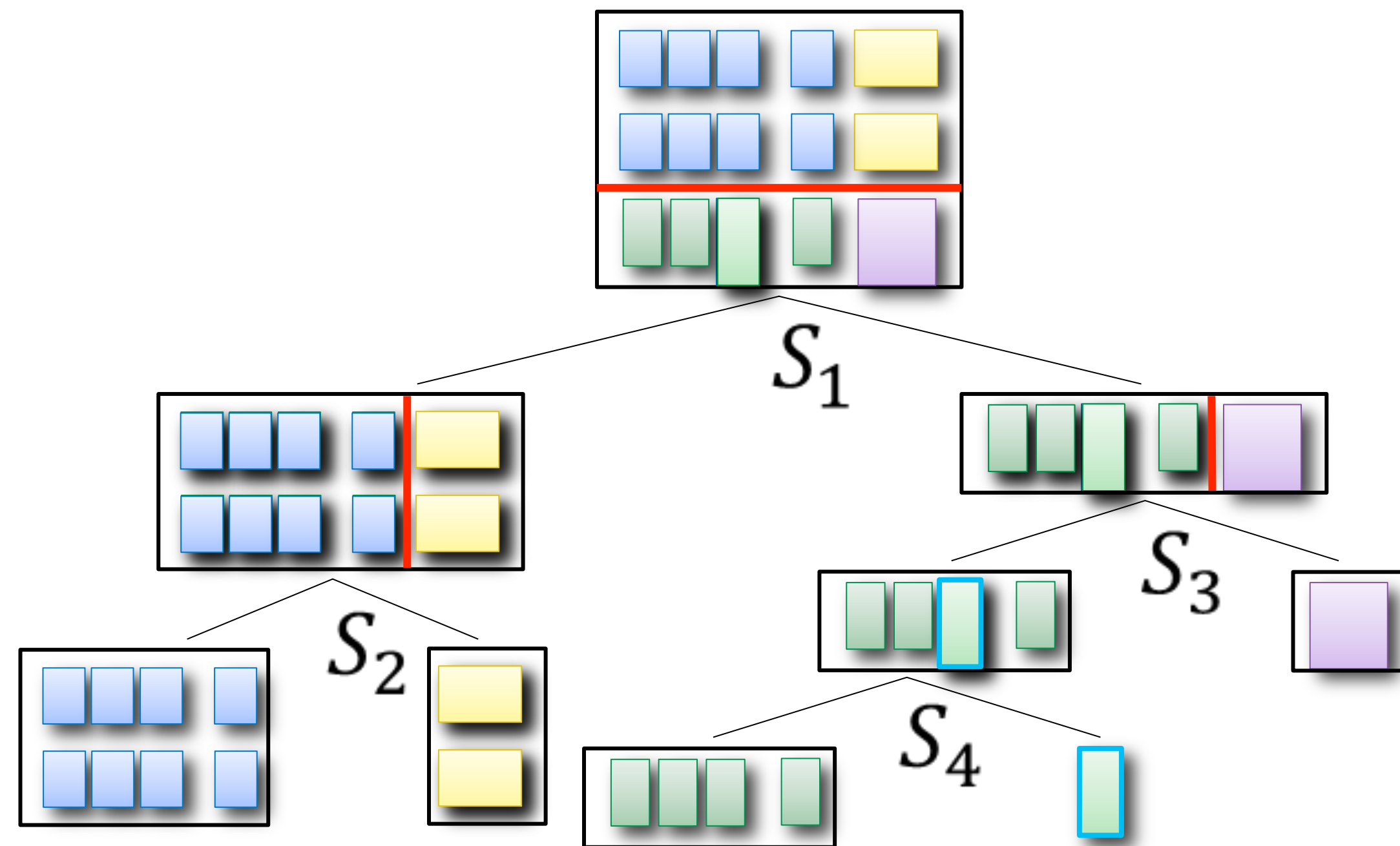


# Symmetry maximization

- Symmetry maximization at each decomposition



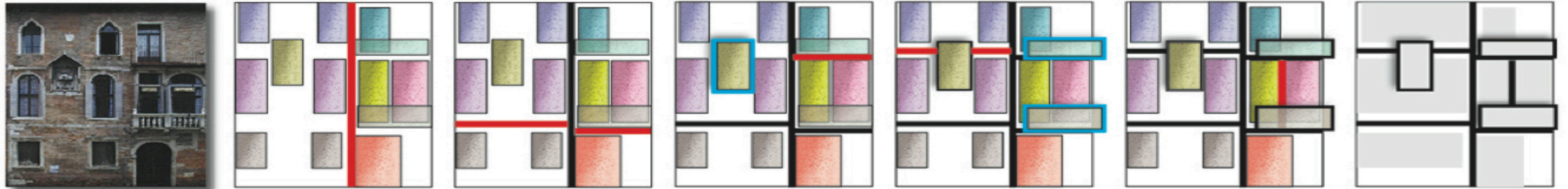
- Sum of **symmetry measures** at all internal nodes



$$\max. \quad \Sigma = S_1 + S_2 + S_3 + S_4$$



# SYMAX vs. symmetry hierarchy



## SYMAX

- involves explicit optimization
- symmetry hierarchy with an objective function

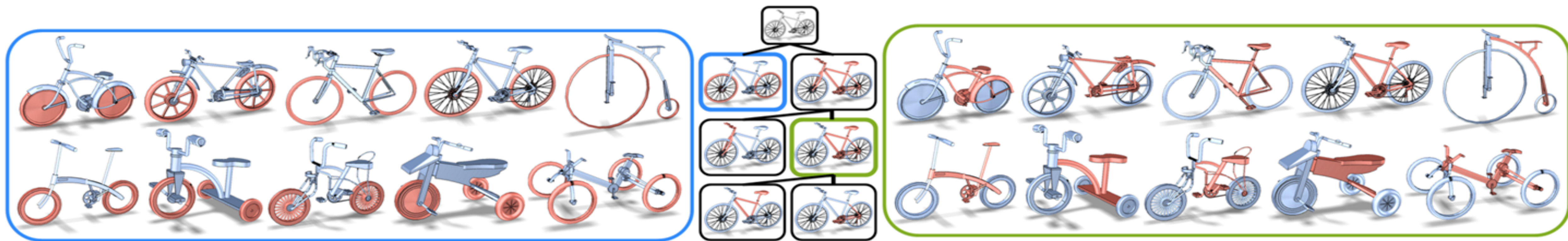
Both still confined to analyzing individual models



# Co-hierarchy [van Kaick et al. 2013]

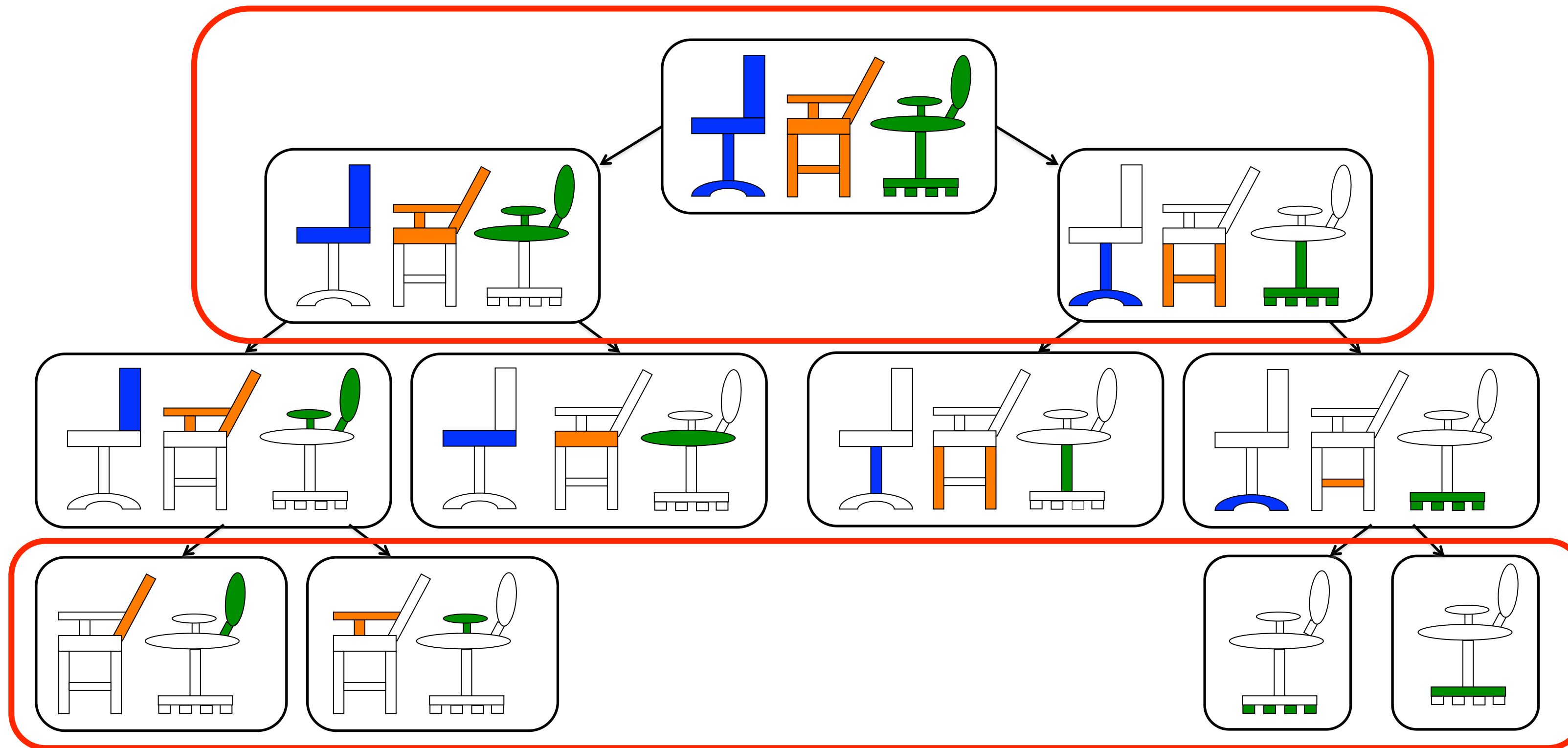
Goal: obtain a structural hierarchy that best explains a **set of objects** belonging to the same class

- Extends symmetry hierarchy to **co-analysis** of a set
- A **unified explanation** of the shape structures

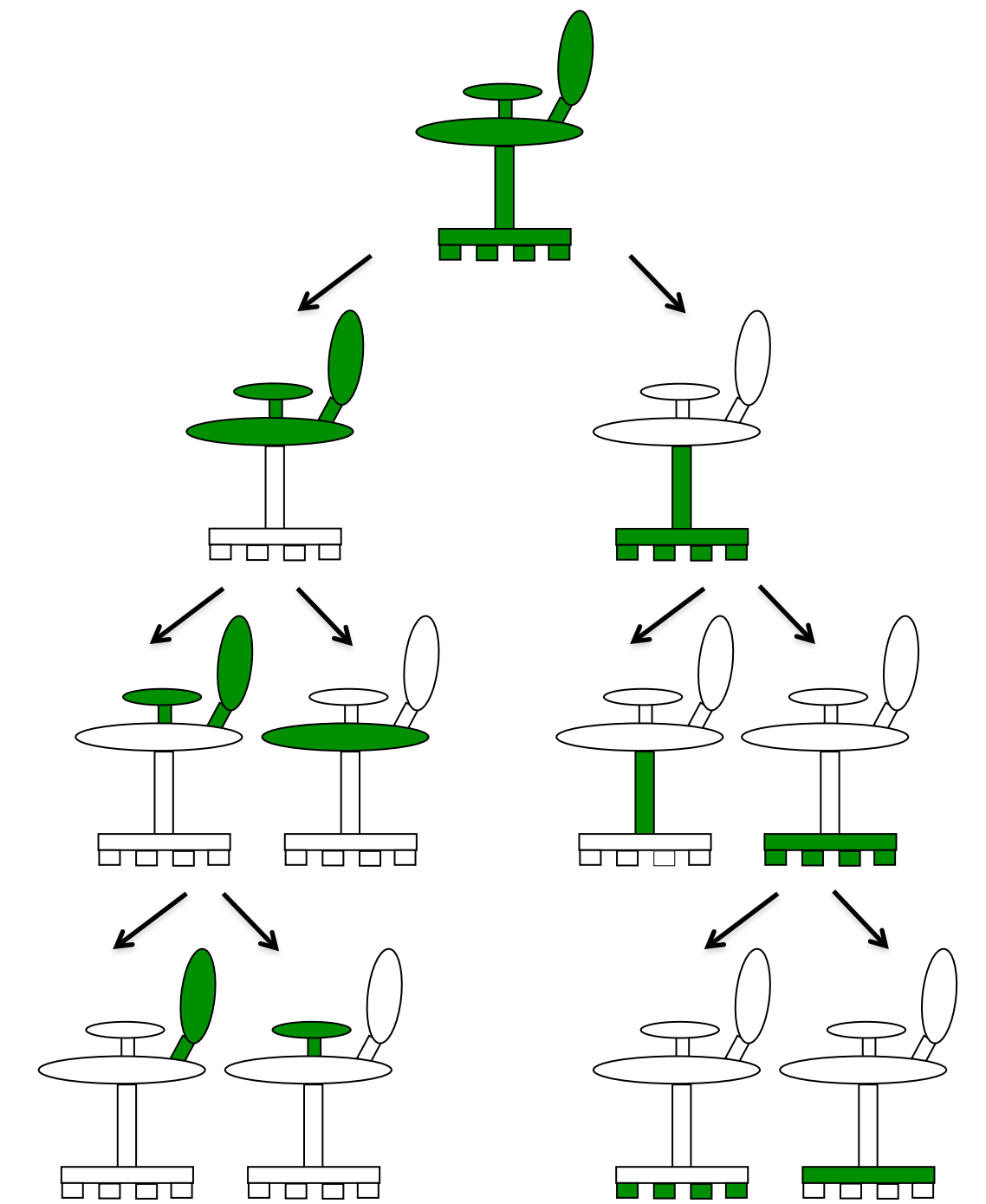


# Similarity and diversity

- **Coarse level:** similarity across set

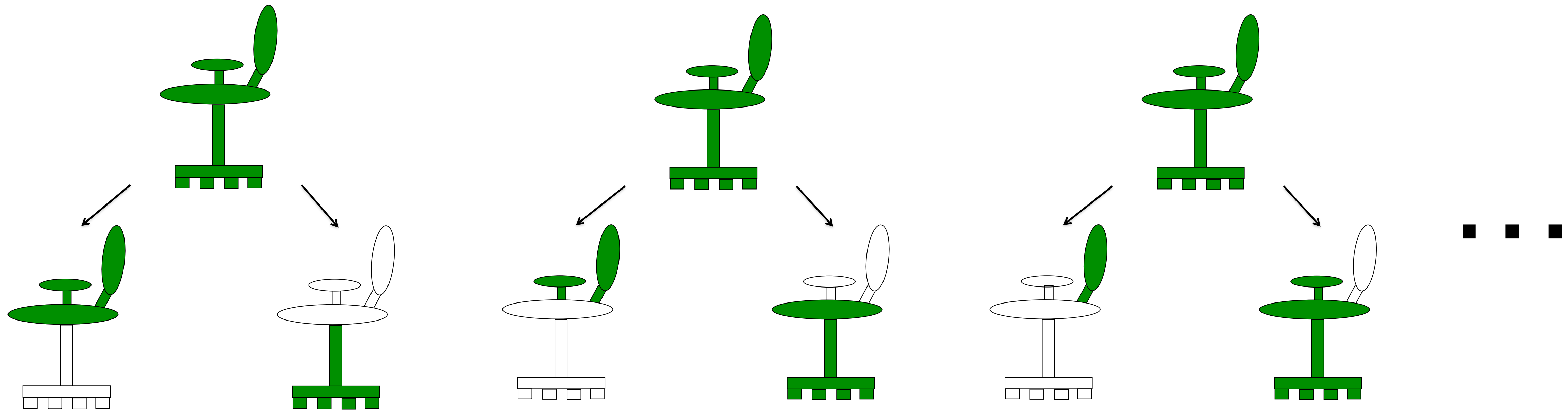


- **Finer level:** individual shape variations





# Challenges



Each object can have **many possible** hierarchies.

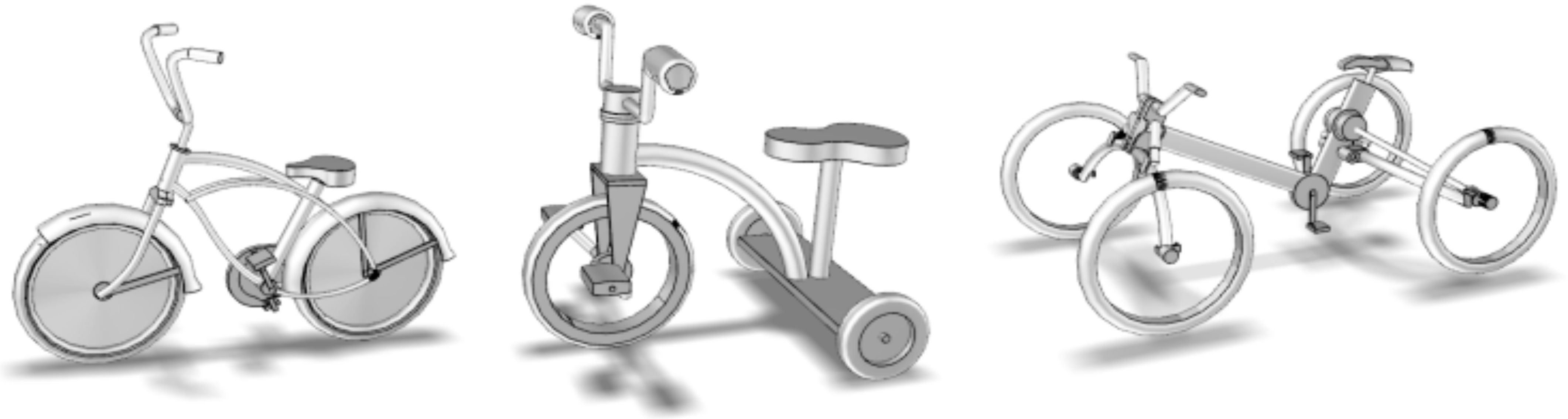
Need to **select** one hierarchy per shape.

# Challenges



There can be much **geometric variability** in the set.  
Need to go beyond geometry and focus on shape **structures**.

# Challenges

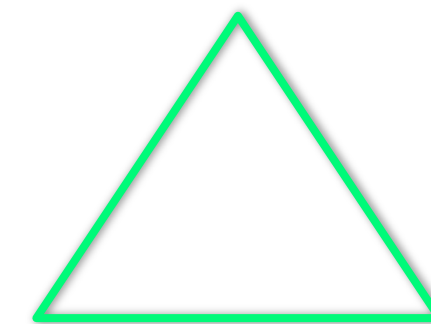
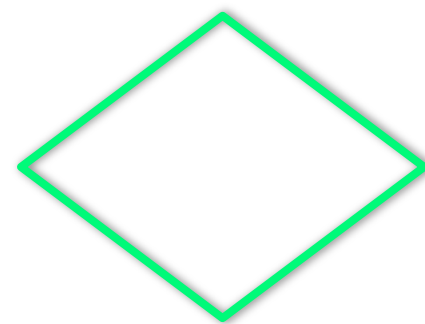
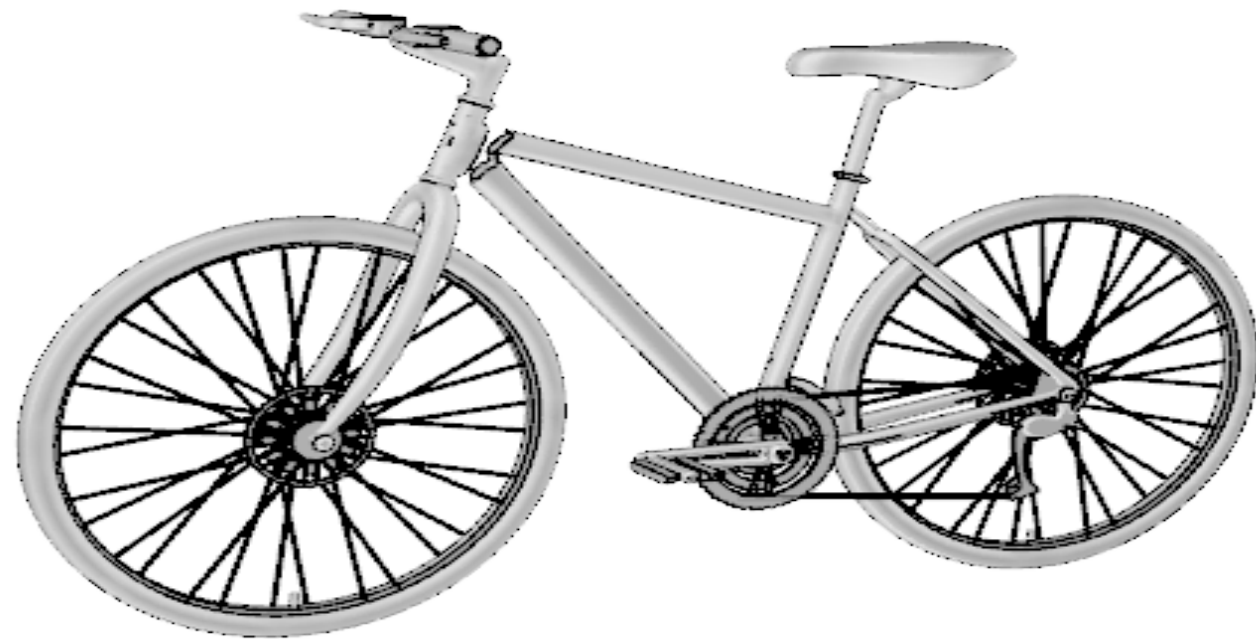
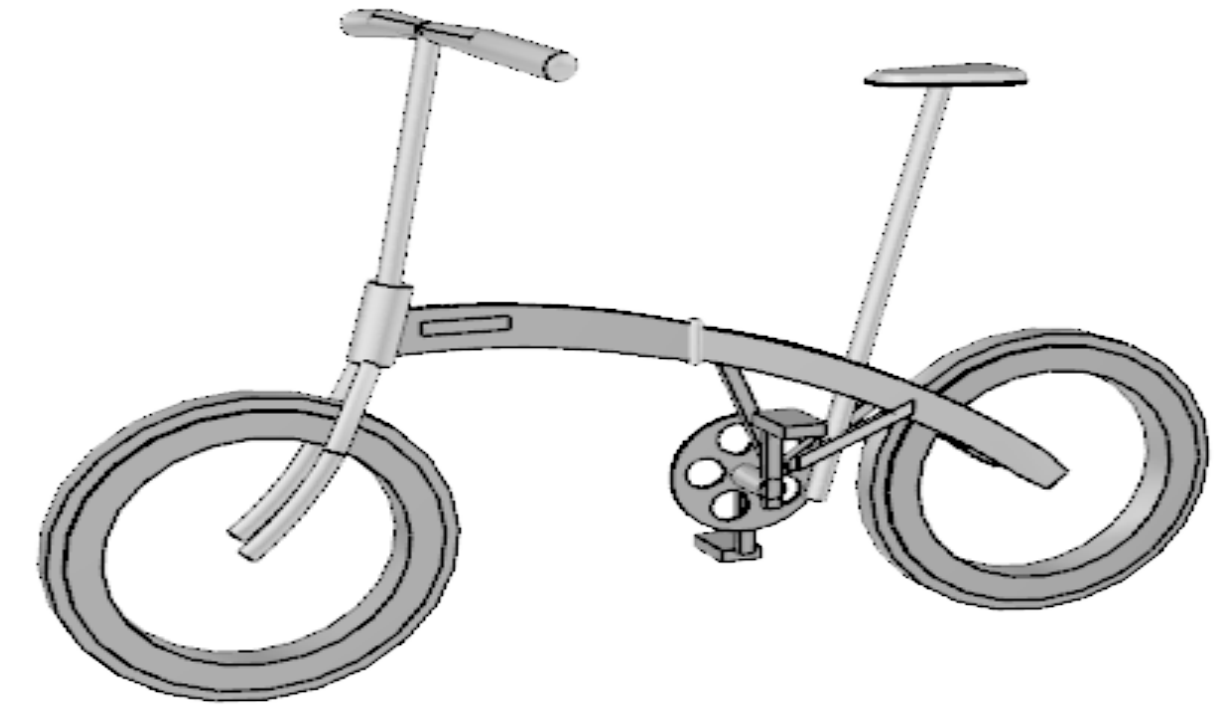
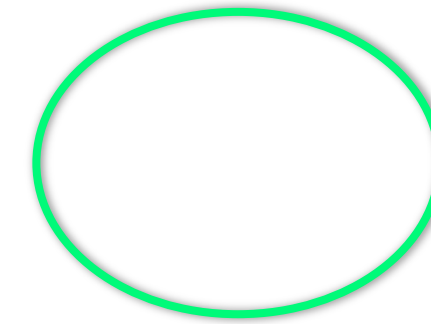
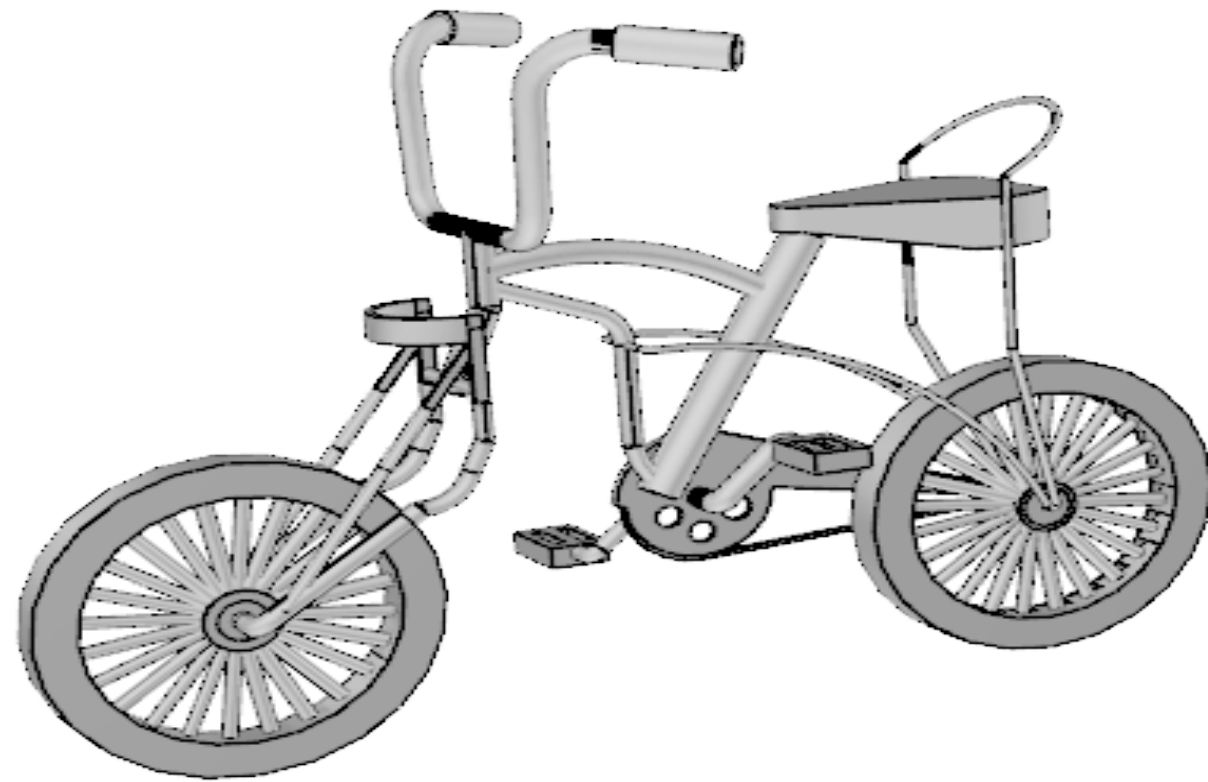


There can also be much **structural variability**; a single explanation cannot be sufficient

We account for that by **clustering** the hierarchies.

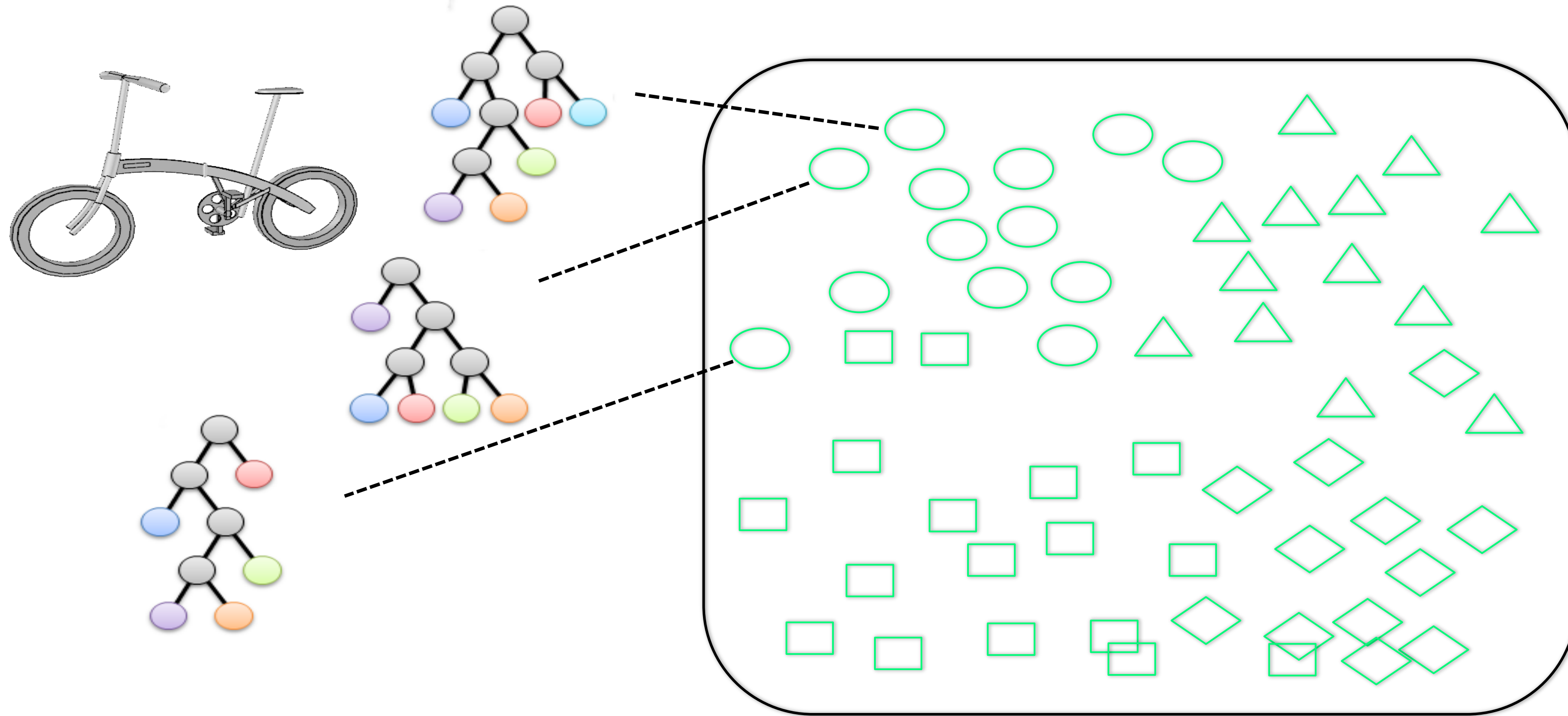


# Algorithm illustration



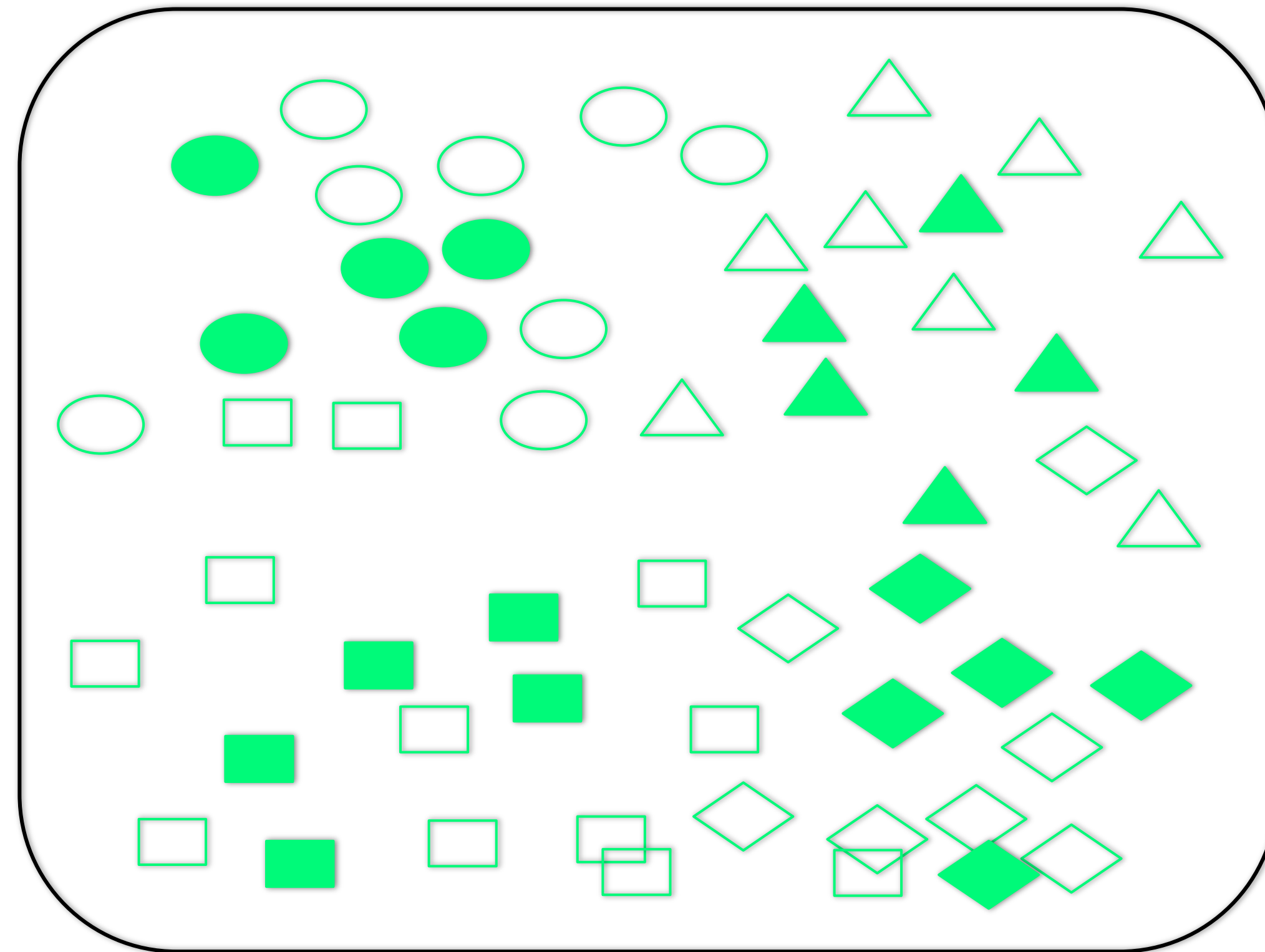
Using a minimal set of four shapes

# Per-shape symmetry hierarchies



Compute symmetry hierarchies per shape

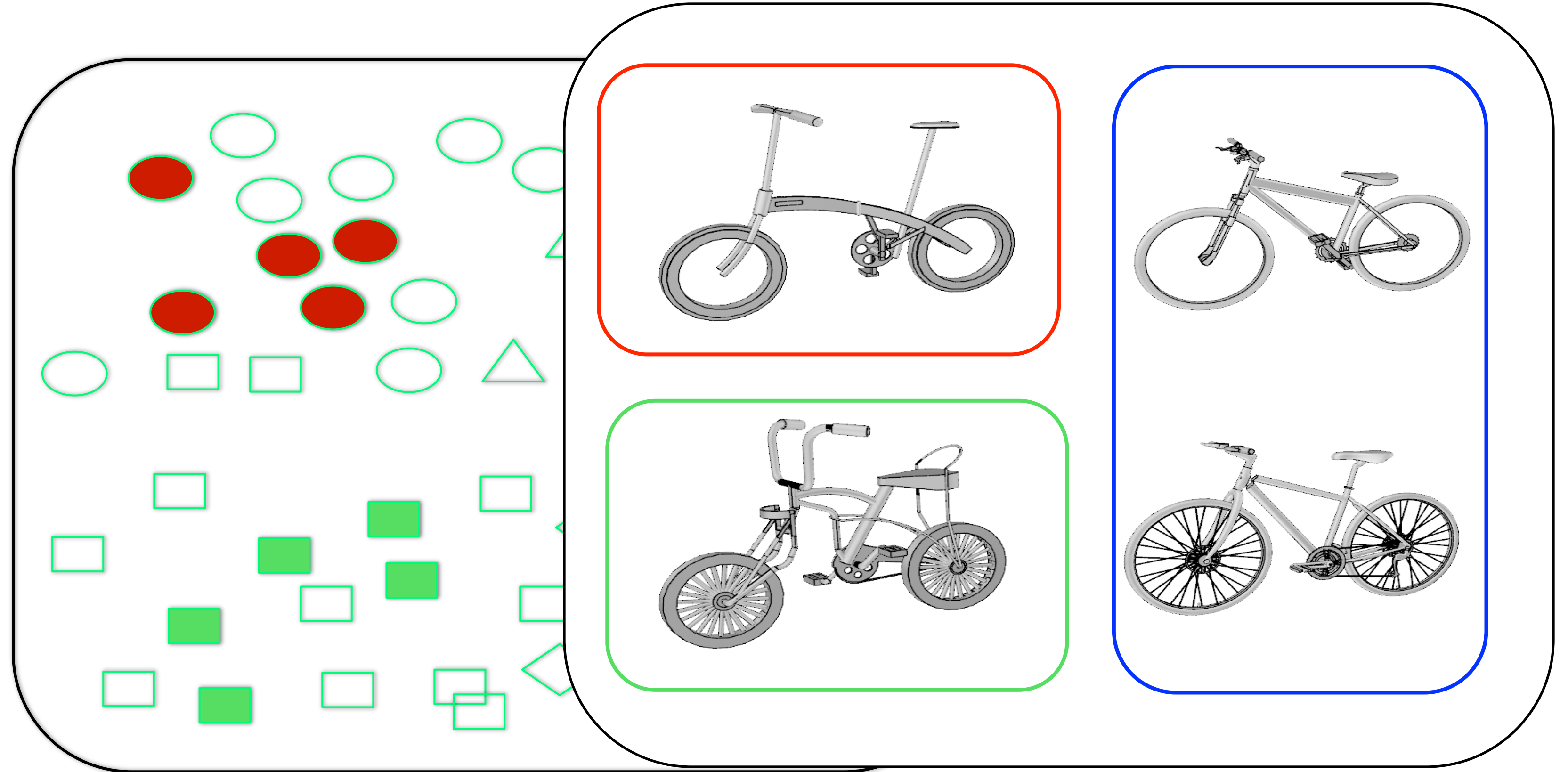
# Sampling



Sample from population of per-shape hierarchies

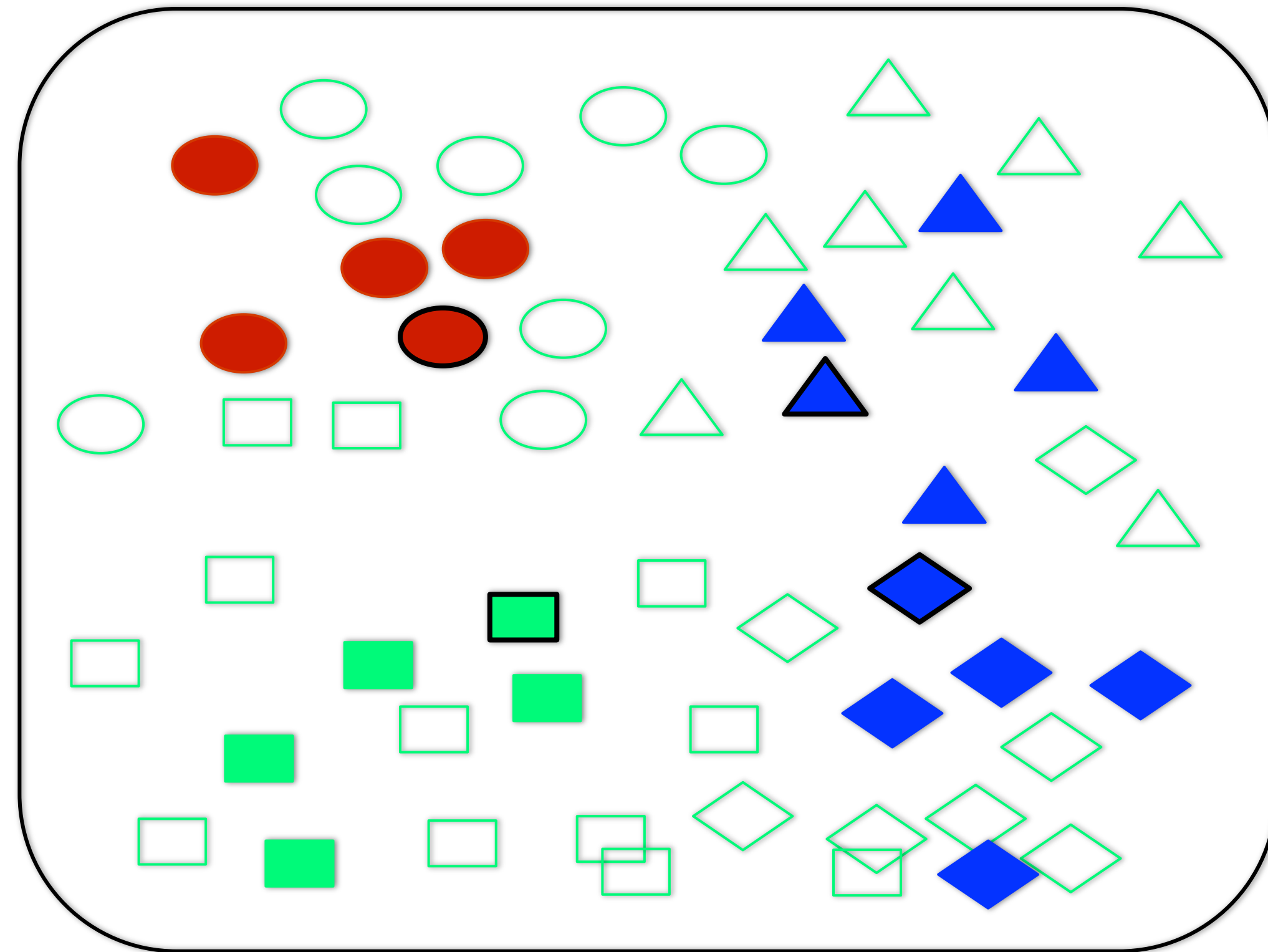


# Key problem: clustering



**Multi-instance** clustering: cluster the shapes while each shape has multiple instances (the samples)

# ... and then select



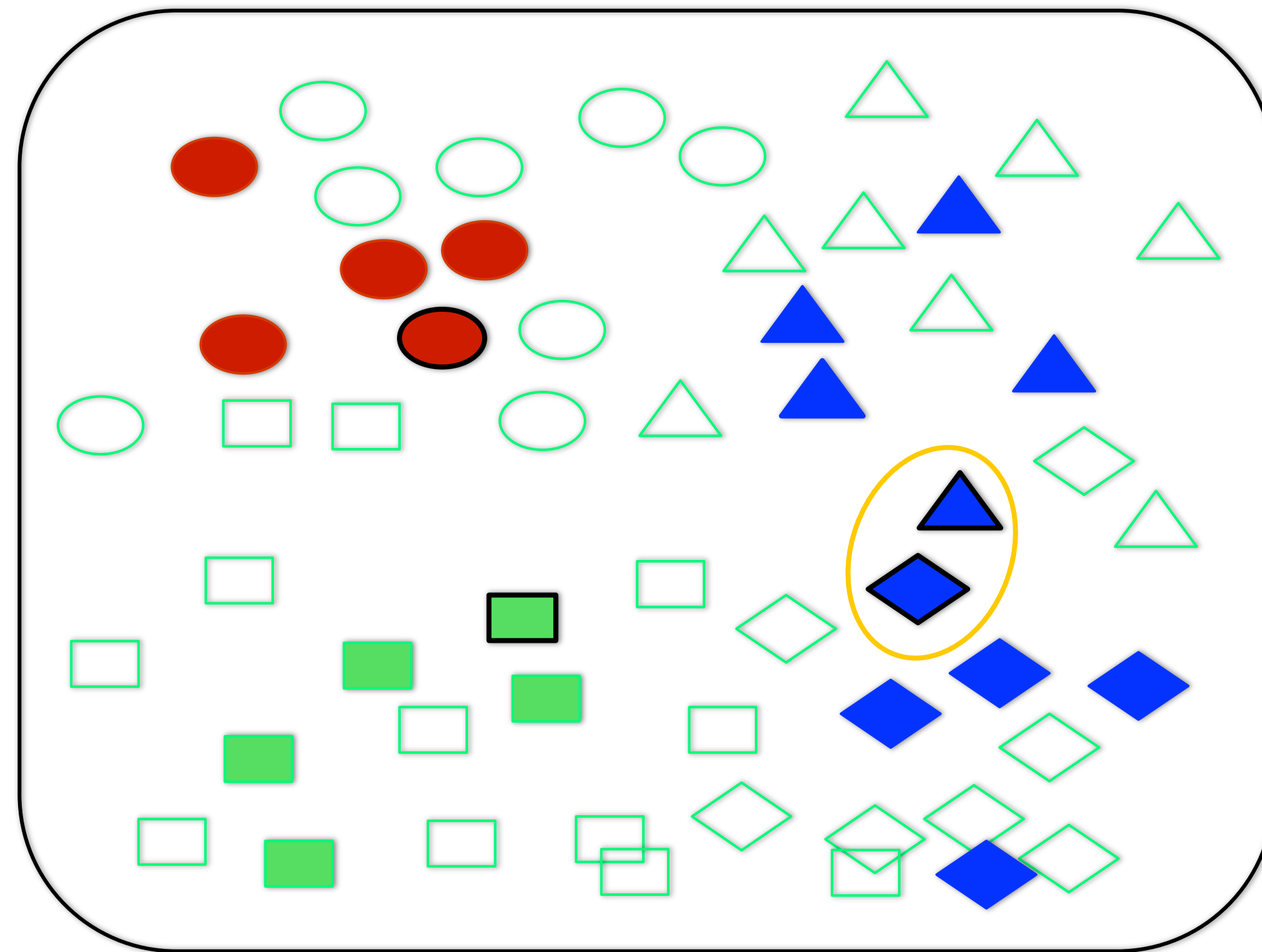
Select **one representative** hierarchy **per shape**

# Objective of cluster-and-select

- Traditional clustering: maximize within-cluster similarity and between-cluster **dissimilarity**
- In our problem: all shapes in the set are related
  - So wish to discover maximal **similarity among all shapes**
  - But still allow clusters to characterize structural variability
  - Trick in **representative selection**: selections should **maximize both within-cluster and between-cluster similarities**
  - Then **resample hierarchies (and then cluster-select) close to the selections** so the clusters as a whole also get closer

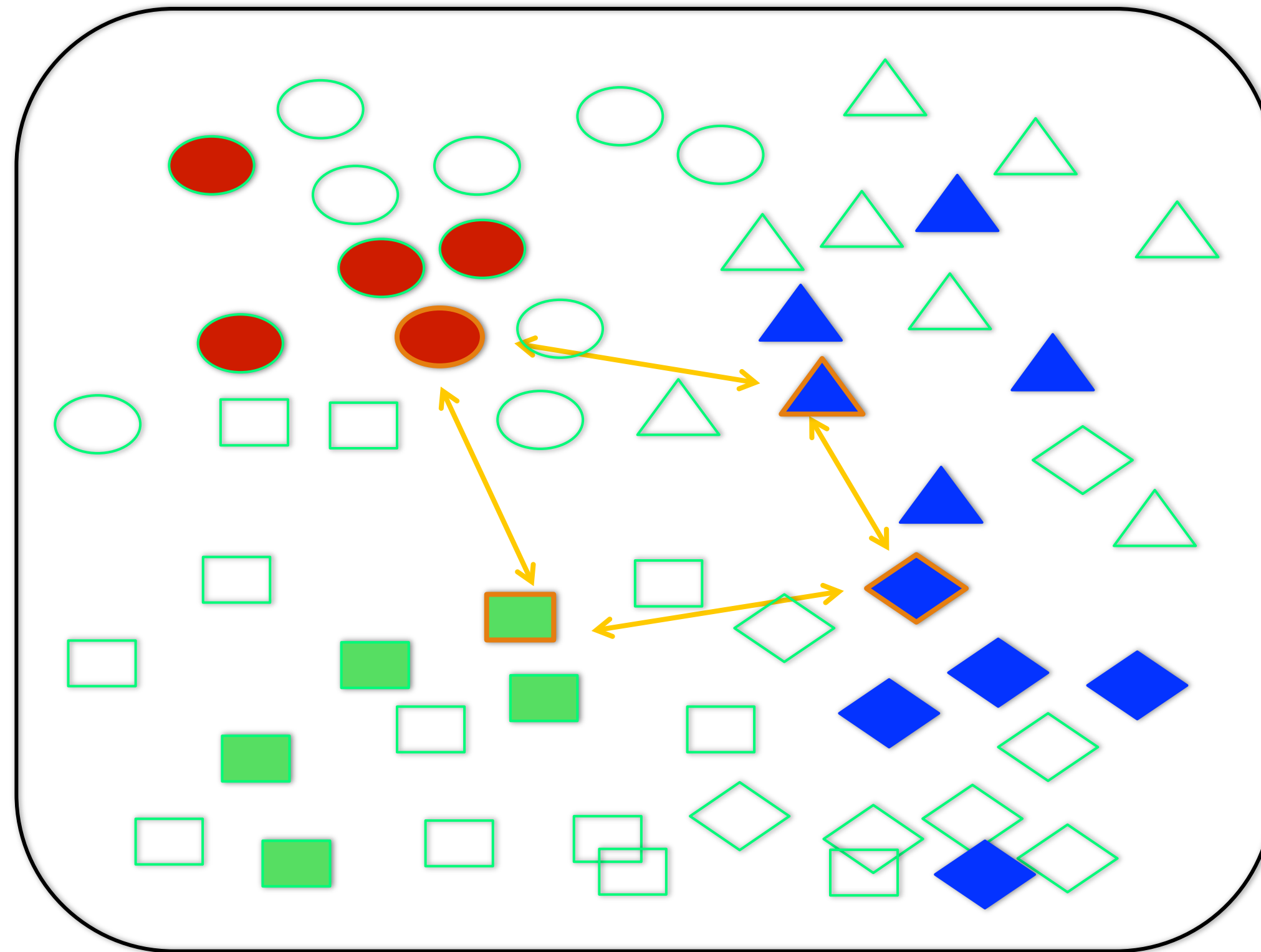


# Representative selection



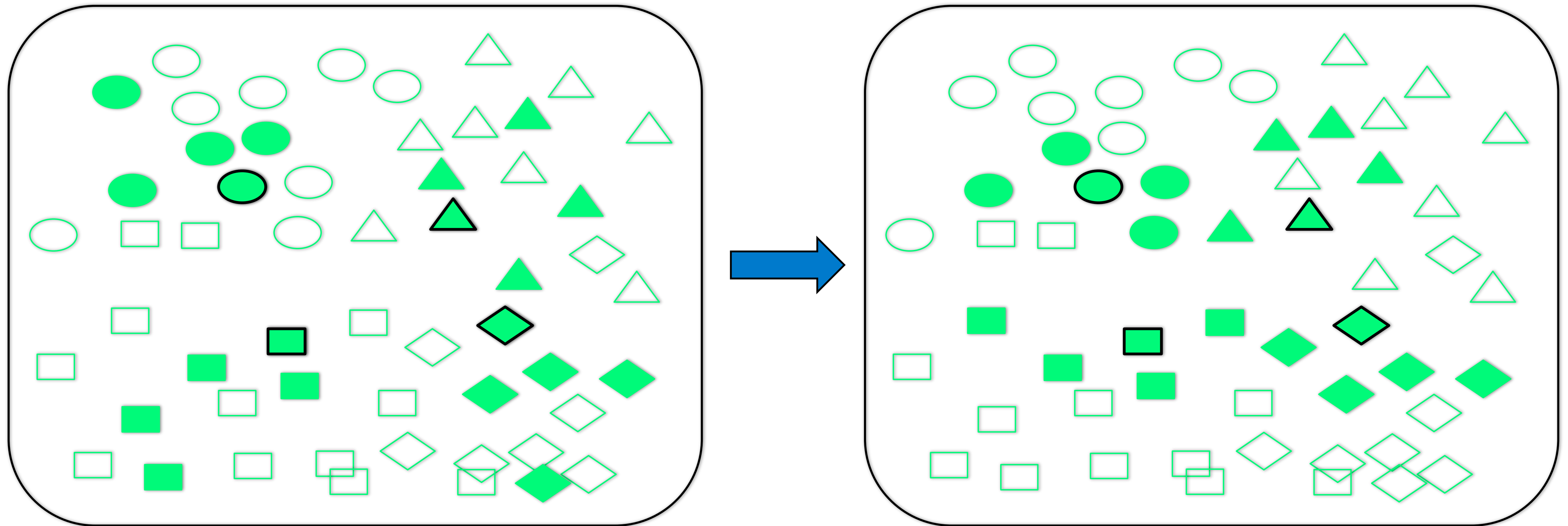
Selections maximize within-cluster similarity only

# Representative selection



Selections maximize both within- and between-cluster similarity

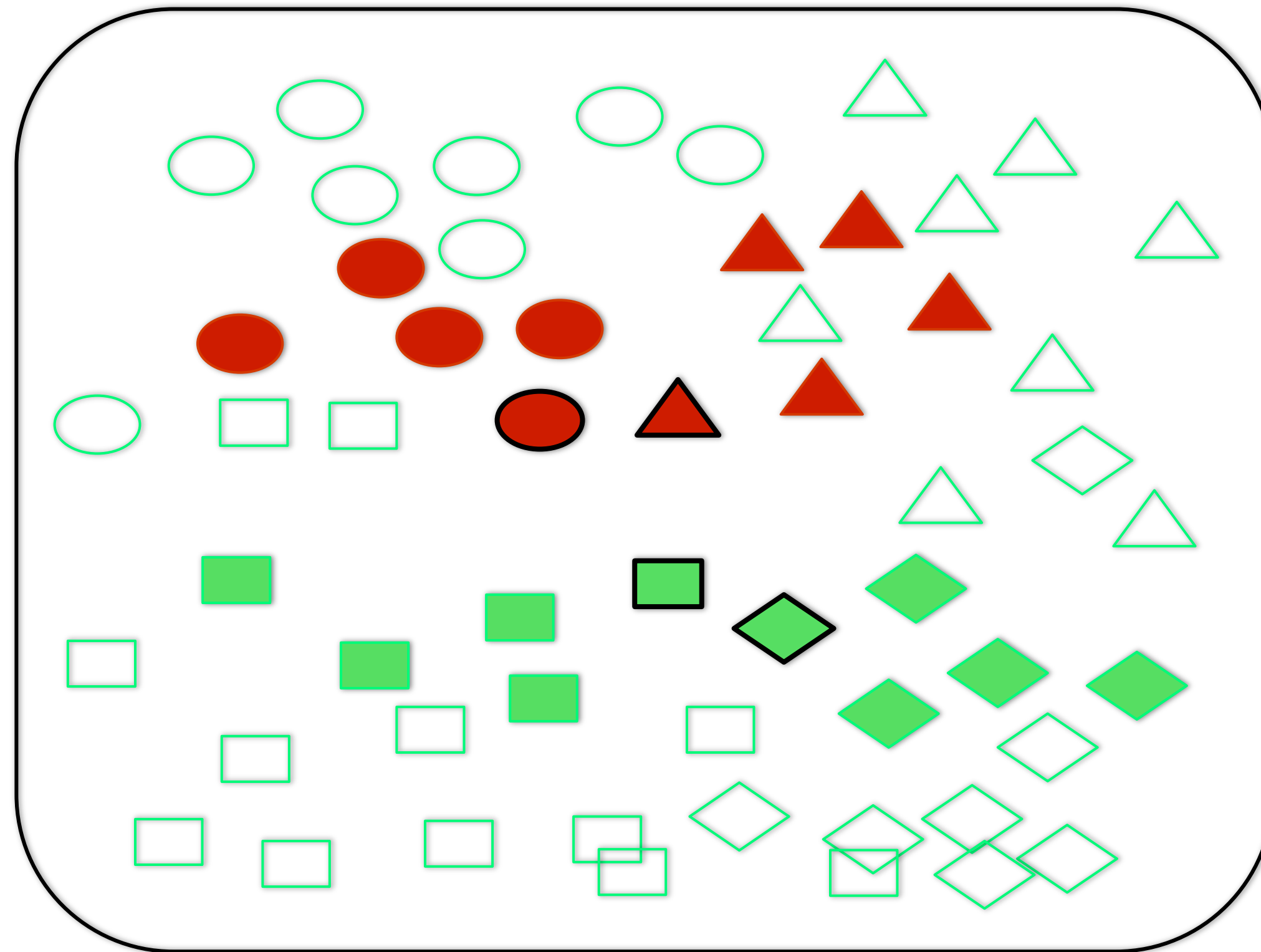
# Resample per-shape hierarchies



New samples are closer to the representatives



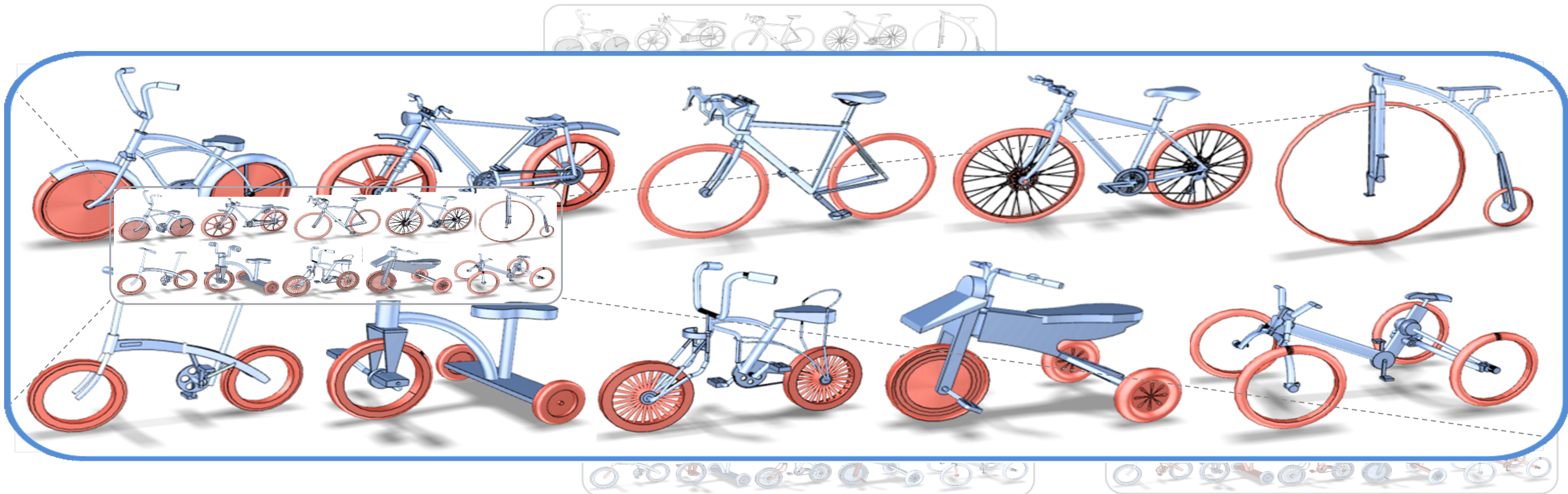
# Iterate cluster-select-resample



New cluster-select result and repeat until convergence.



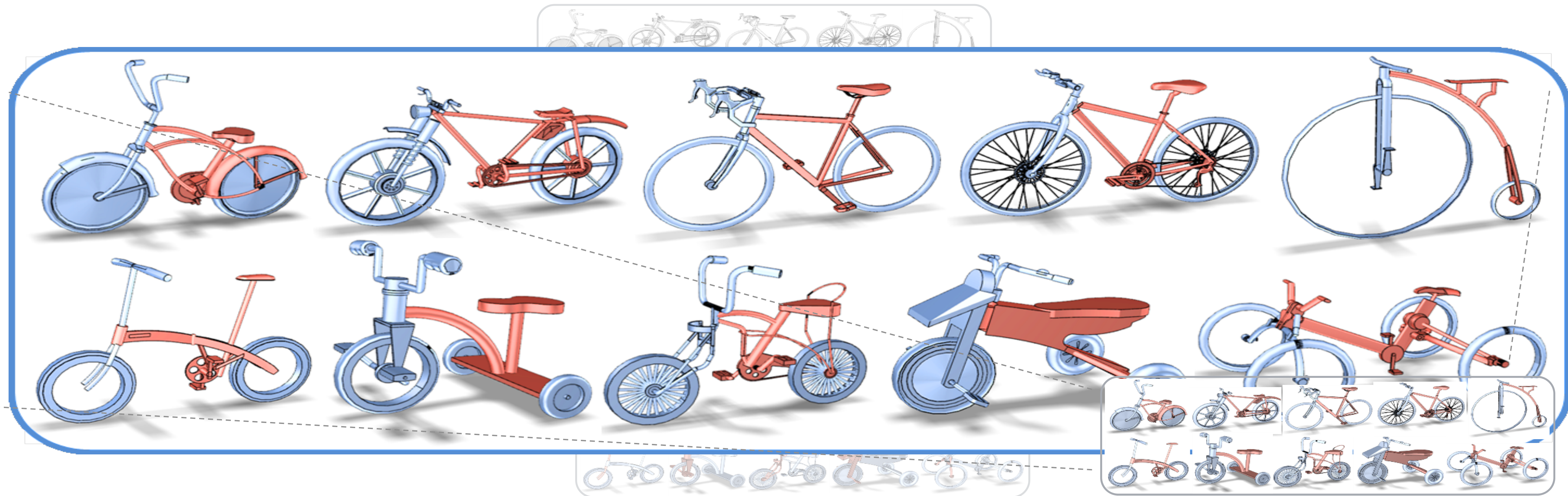
# Structure correspondence



A co-hierarchy implies correspondence between structures ---  
towards **functional correspondence**.



# Functional correspondence



A co-hierarchy implies correspondence between structures ---  
towards **functional correspondence**.



# A short “now and beyond”

- Still most works deal with flat structural organizations
- There is much to be done on hierarchical analysis:
  - All covered techniques are unsupervised
  - How about **supervised**: providing symmetry measures is hard; ground truths for and comparison between hierarchical models are not easy
  - Extension from 3D objects to more complex data, e.g., **indoor scenes**, and mixed object categories

# We welcome your feedback!

<http://goo.gl/LfVcHA>

