Course Structure

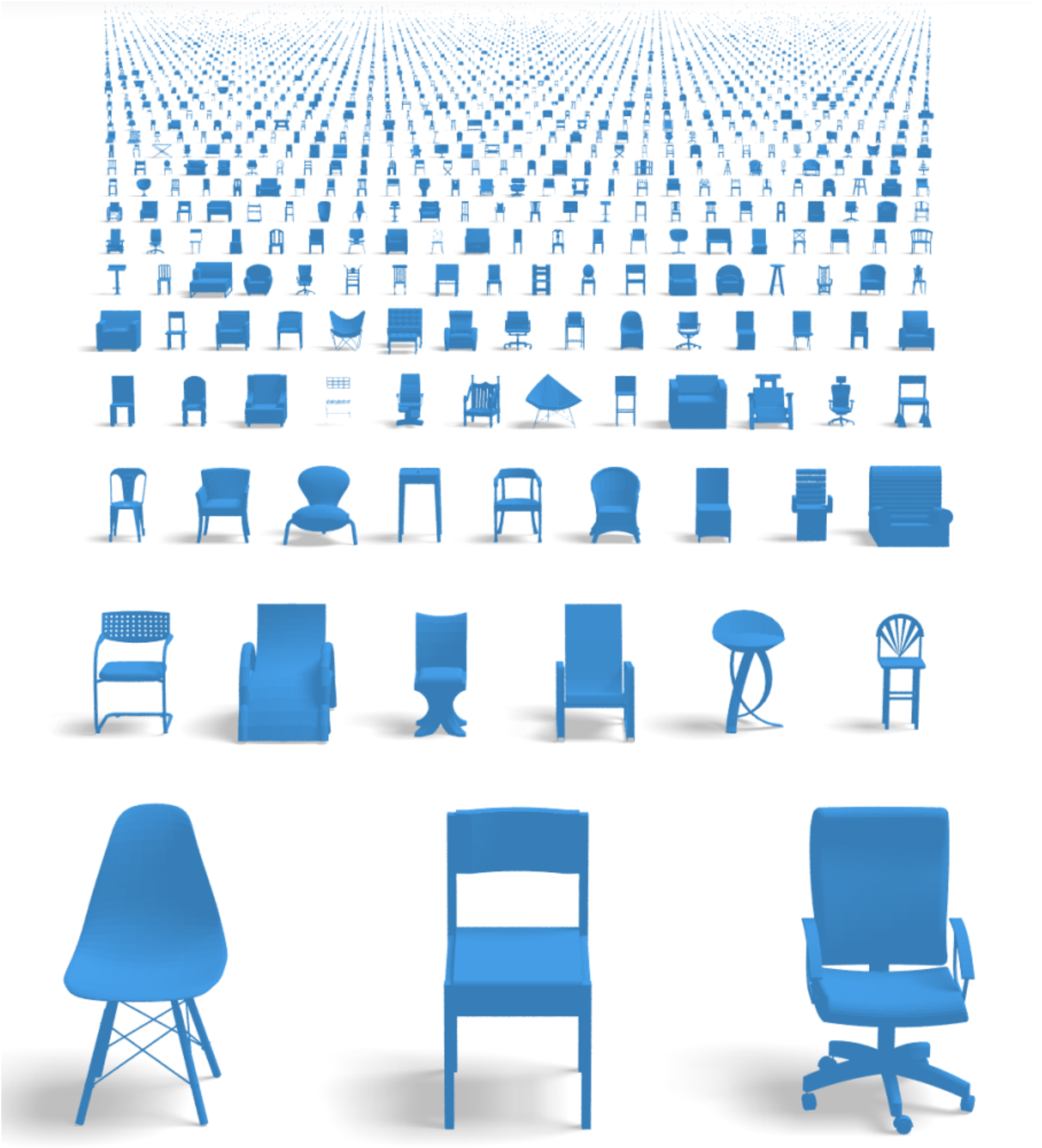
- Introduction to Geometric ‘Structure’
- Extracting Structures
  - analysis of Individual Models
  - **analysis of Shape Collections (co-analysis)**
  - encoding Structural Hierarchy
- Manipulating Structures
  - Modeling as Structural Variations
  - Structure-guided Design
  - Organization + Exploration of Shape Collections
- Future Directions

Extracting Structures

Co-analysis of Model Collections

Single-shape analysis

Accessible to Big Data



Chairs from 3D warehouse

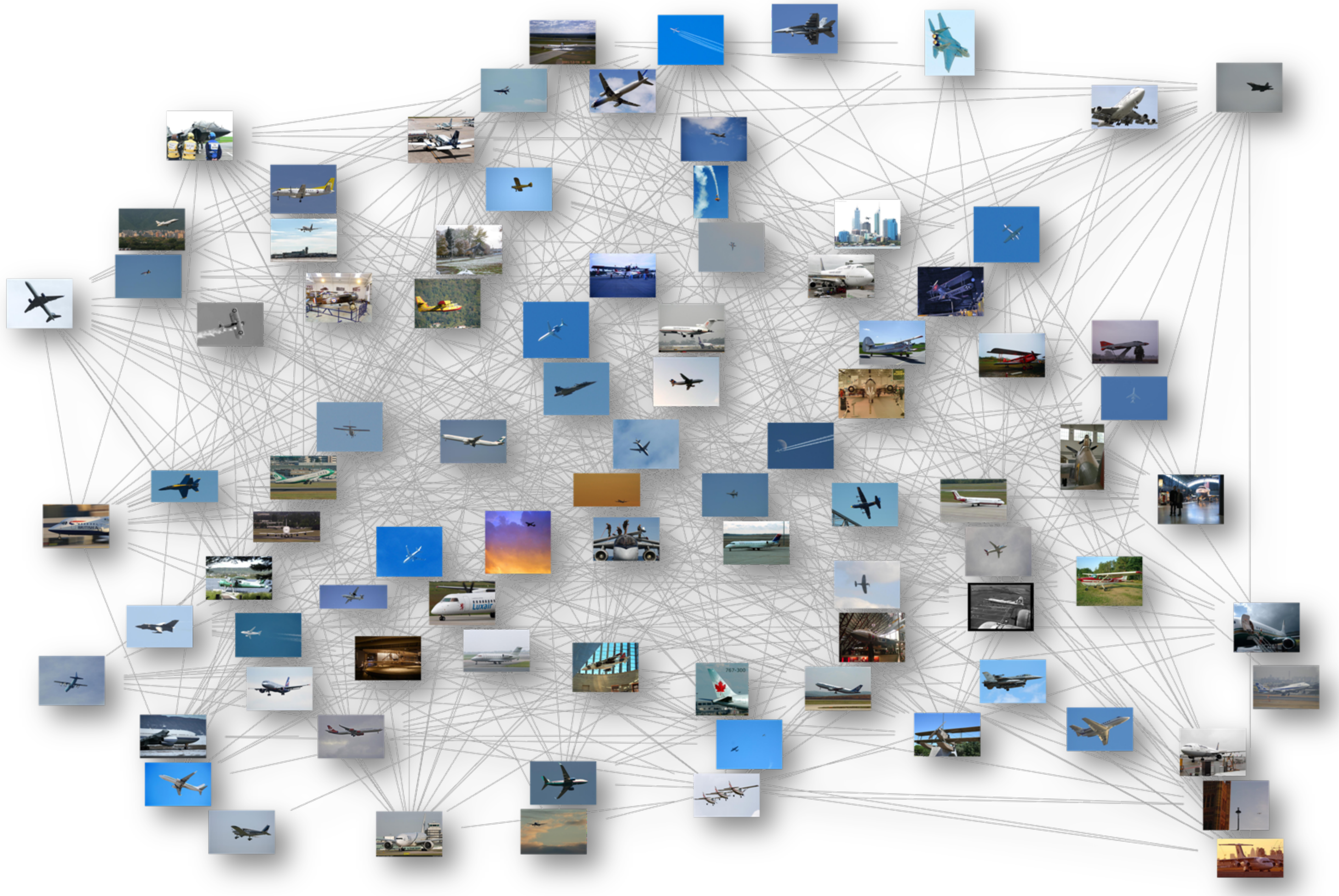
Co-analysis

As we acquire more and more shapes, the shape collections become increasingly interconnected and inter-related, because

- we capture information about the same objects in the world multiple times, or data about multiple instances of an object
- natural and human design often exploits the re-use of certain elements, giving rise to repetitions and symmetries
- objects are naturally organized into classes or categories exhibiting various degrees of similarity

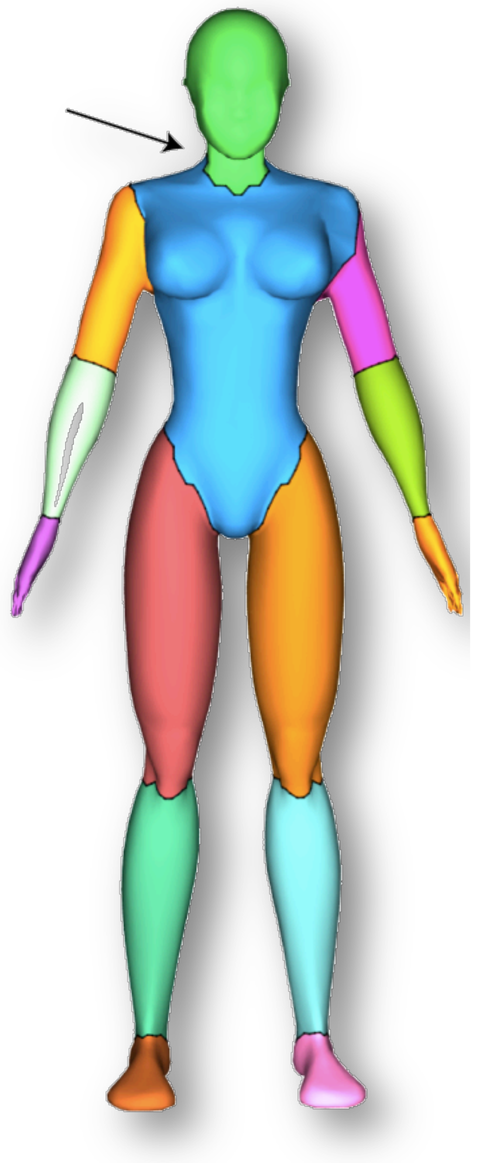
Network view

Co-analysis --- aggregate information from multiple shapes to improve the analysis of individual shapes



Shape segmentation

The interpretation of a shape is deeply influenced by our interpretation of other shapes



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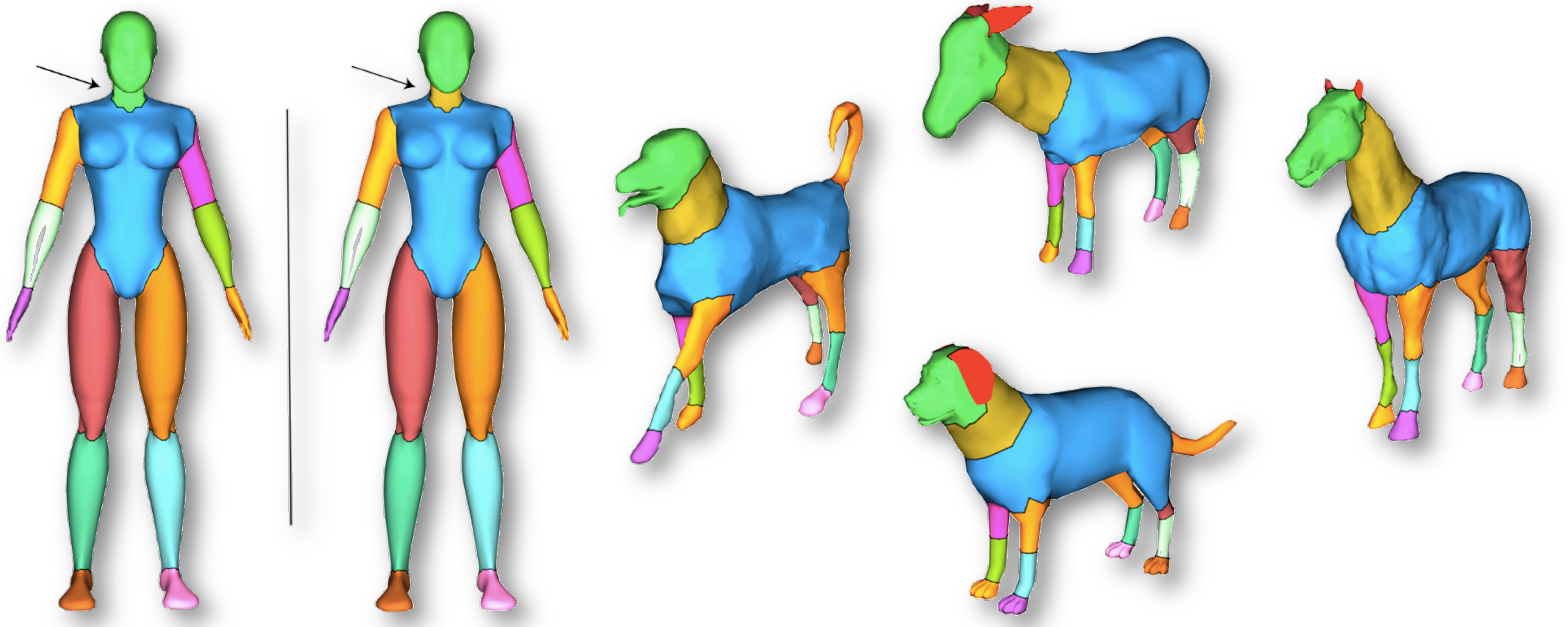
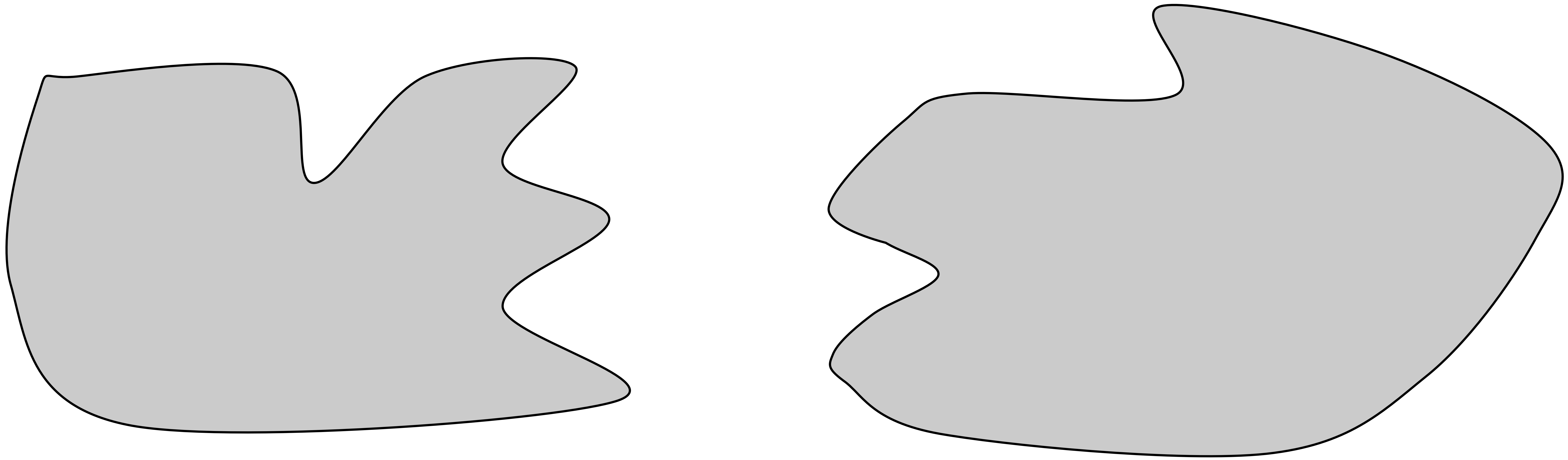
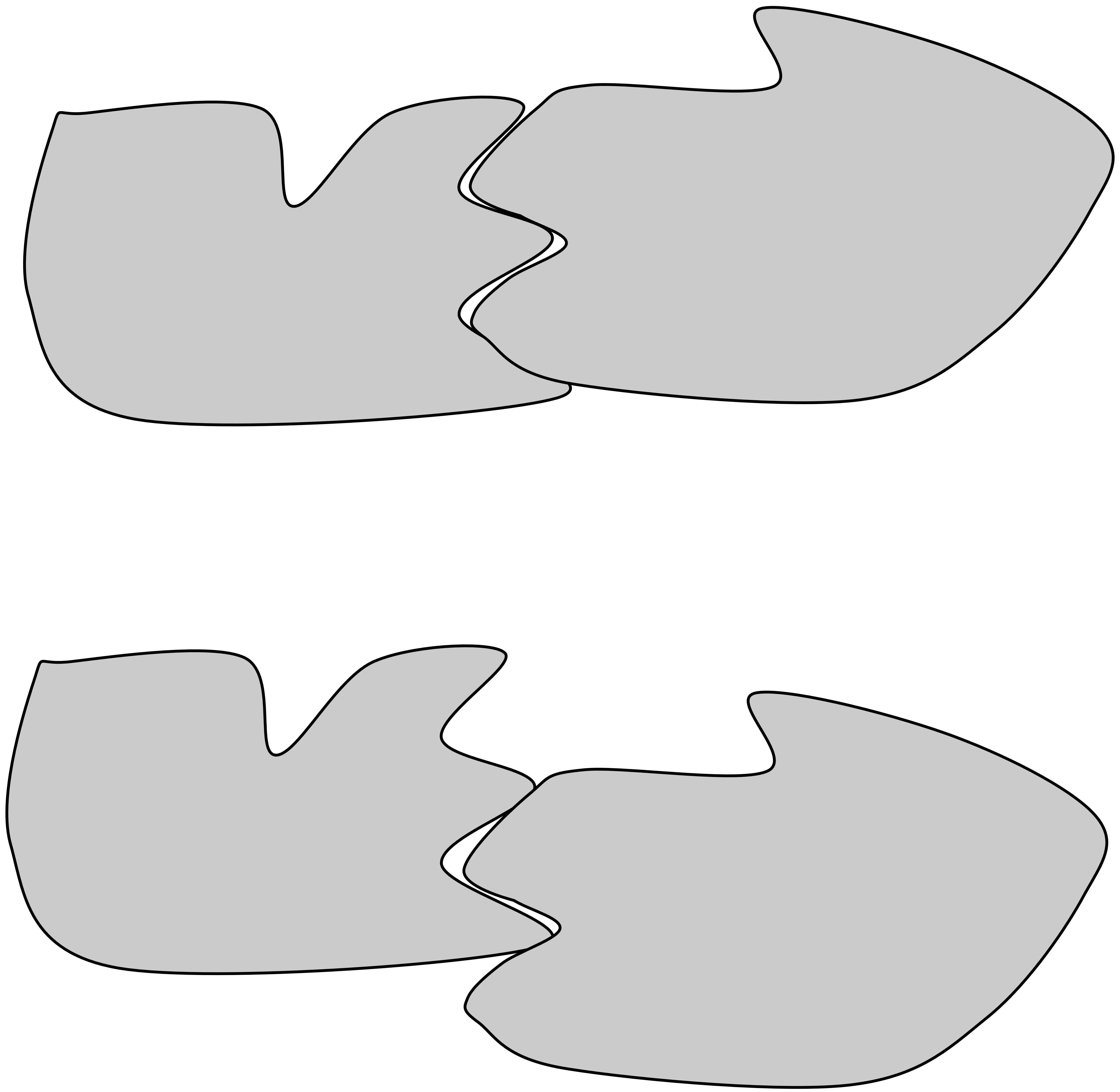


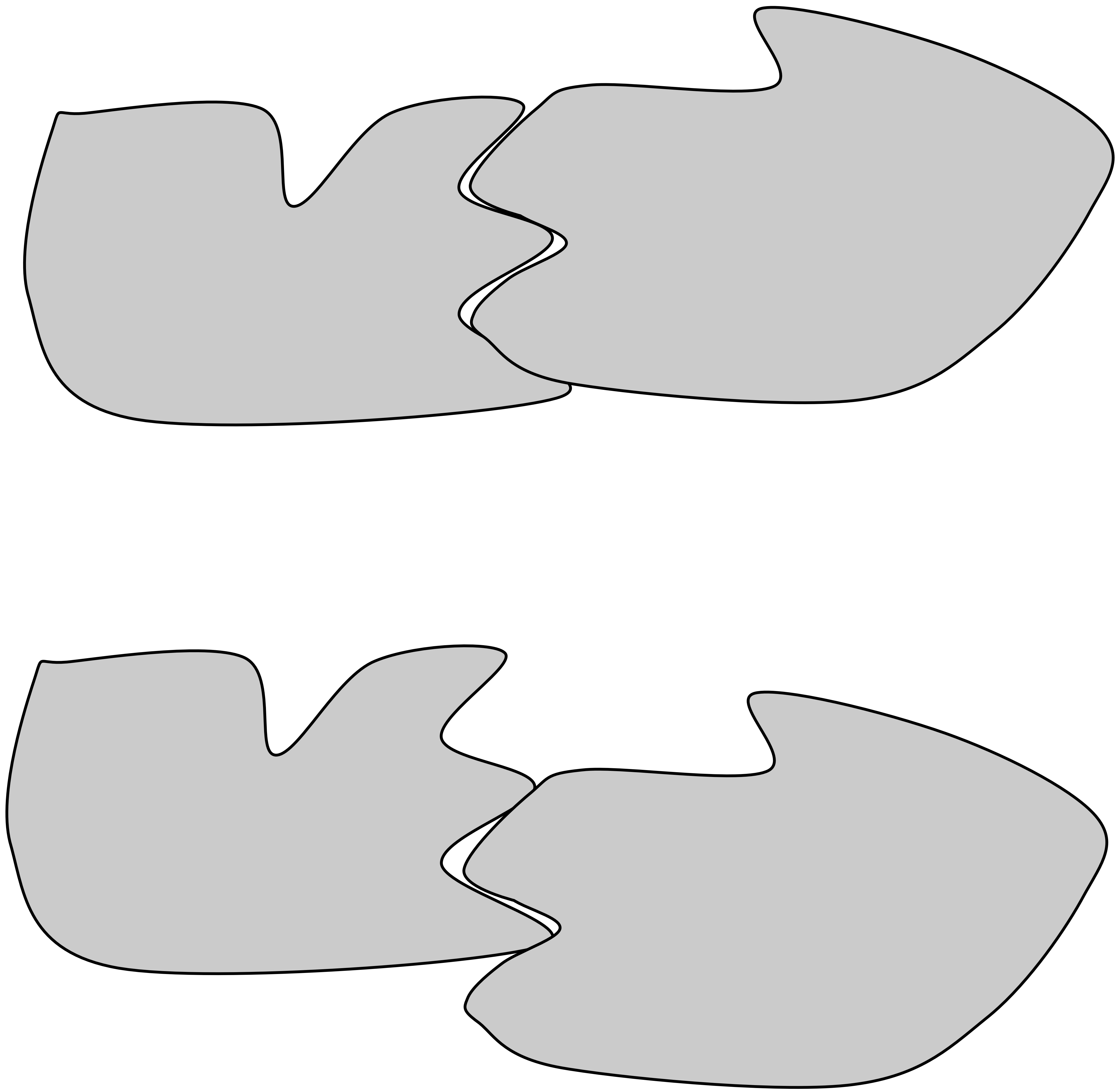
Shape matching



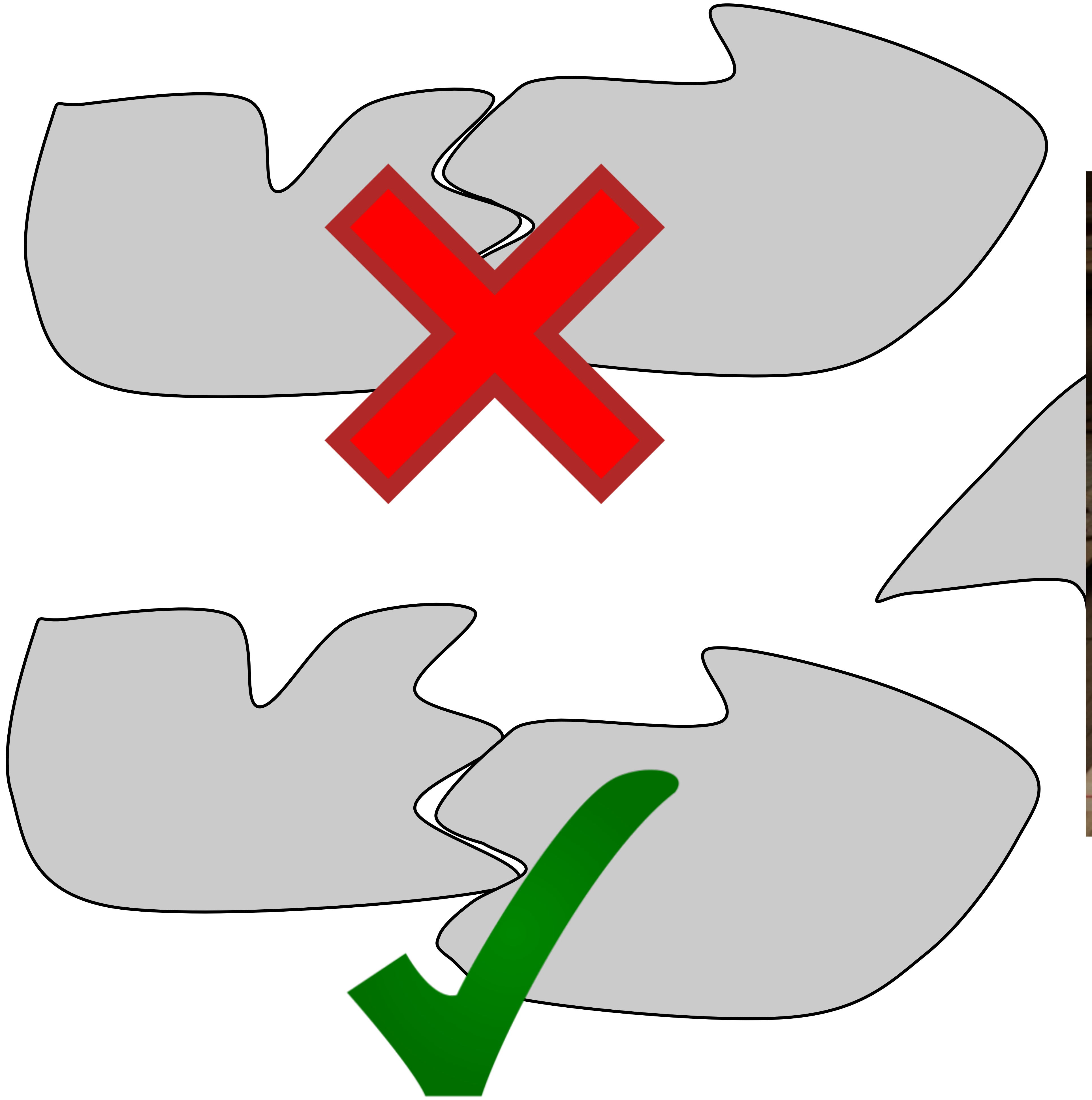
Shape matching



Shape matching

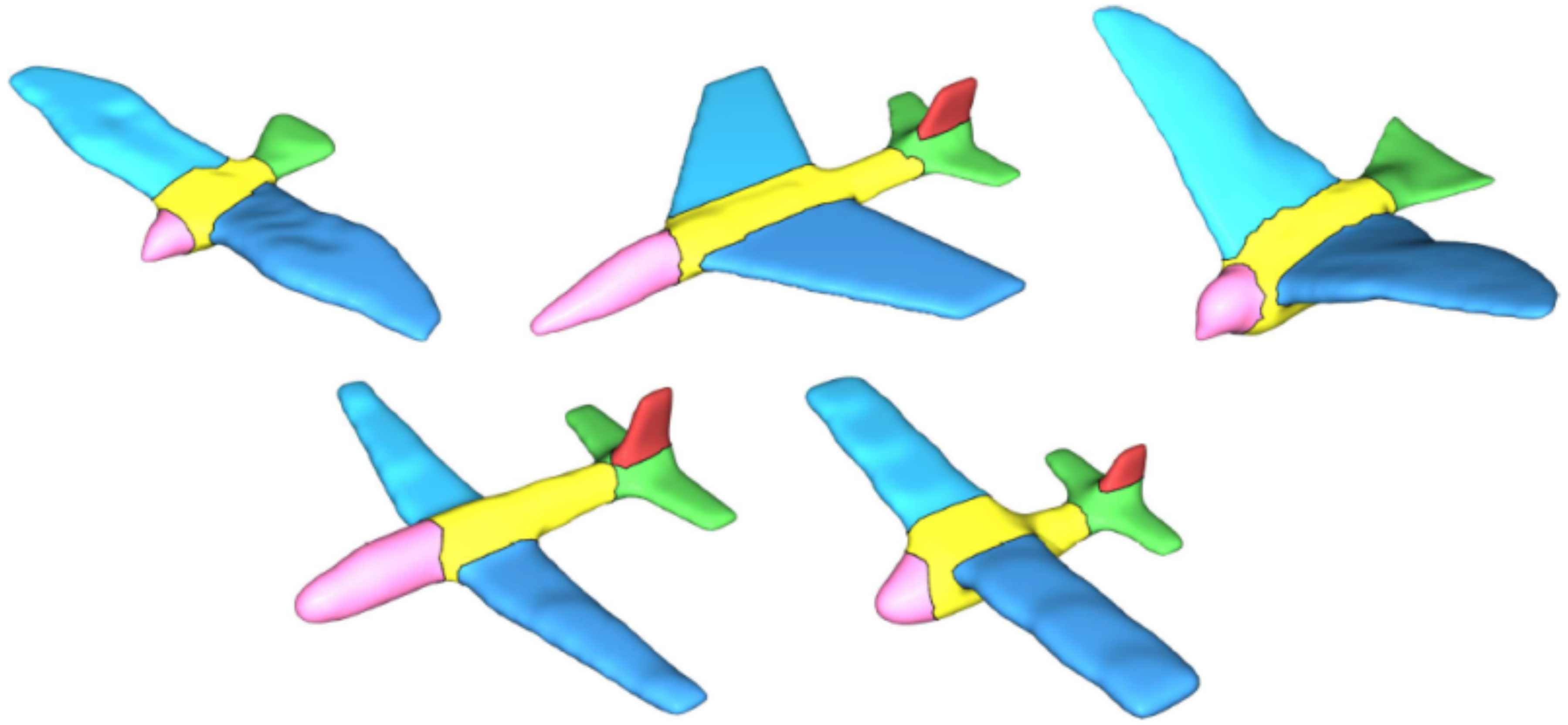


Additional data helps



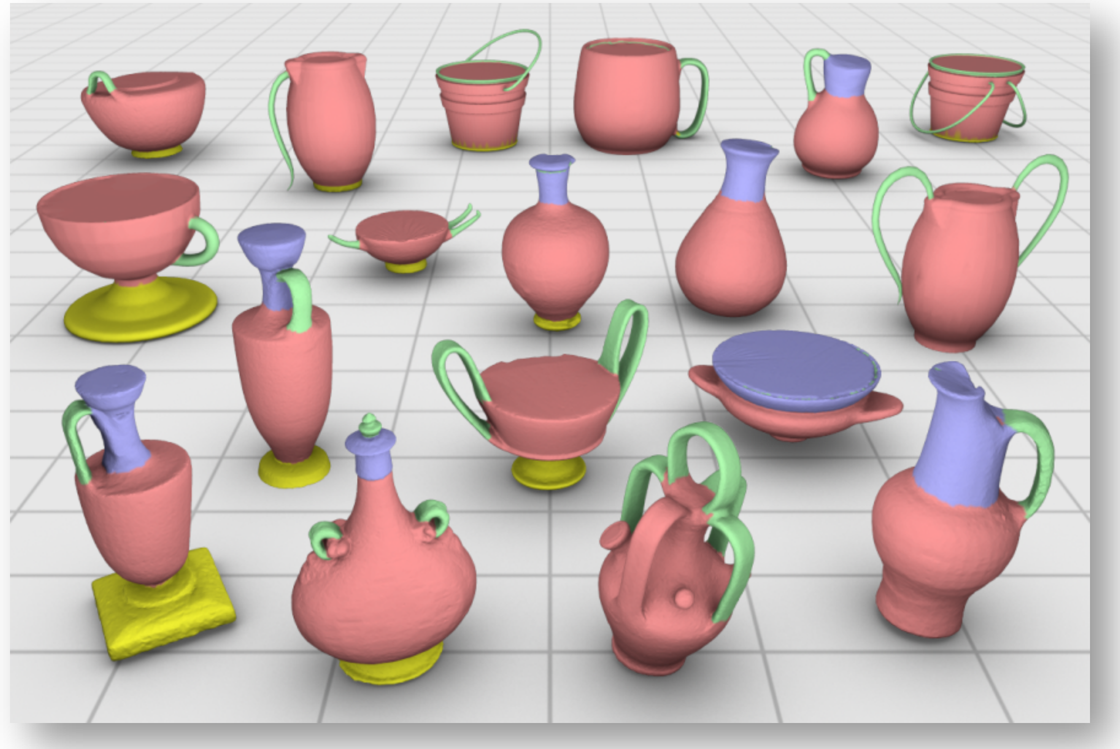
Keys to co-analysis techniques

- Determine the shared information

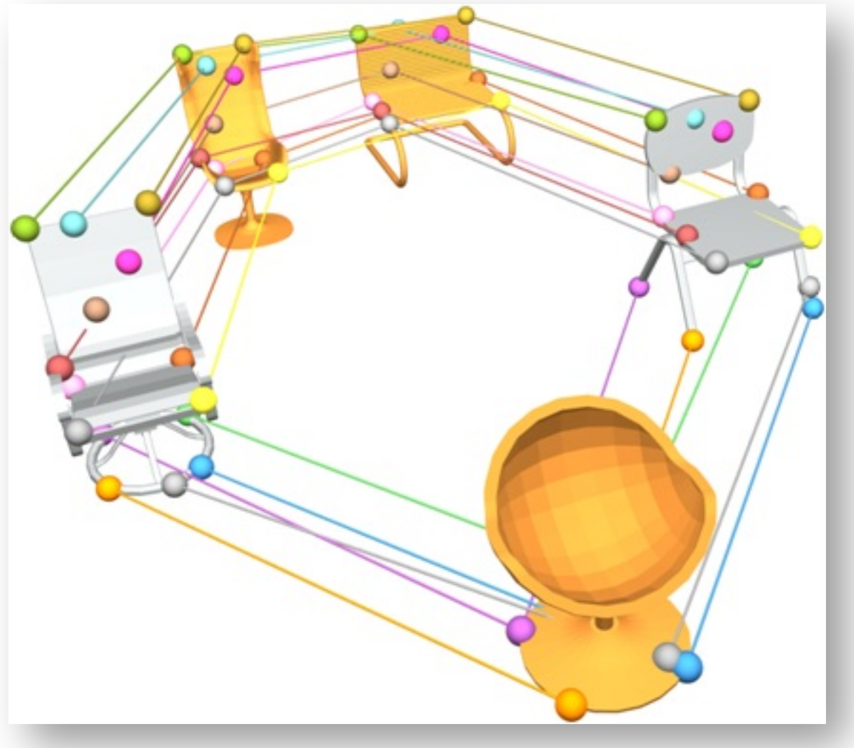


- Methodology (as a subject of data analysis)
  - ❑Supervised
  - ❑Unsupervised
  - ❑Semi-supervised

Outline



Co-segmentation



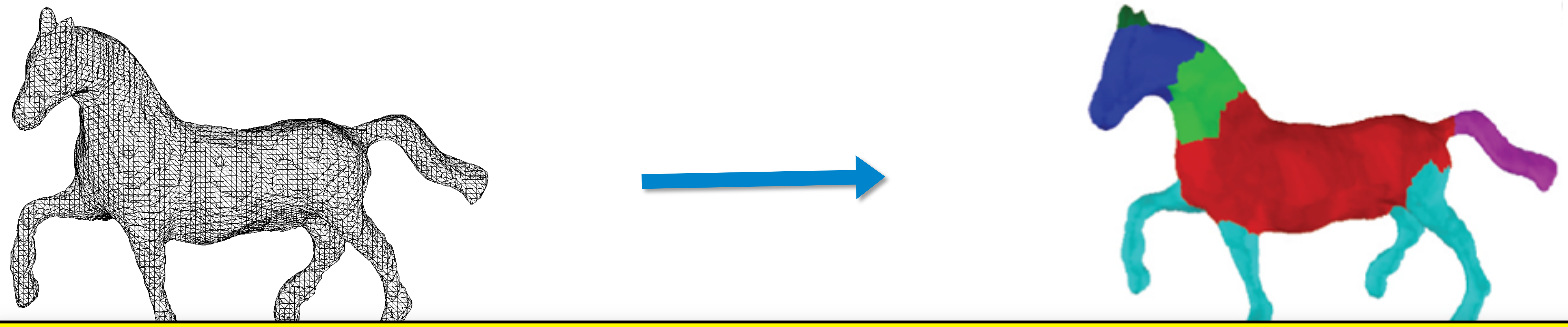
Joint matching

Other types of co-analysis

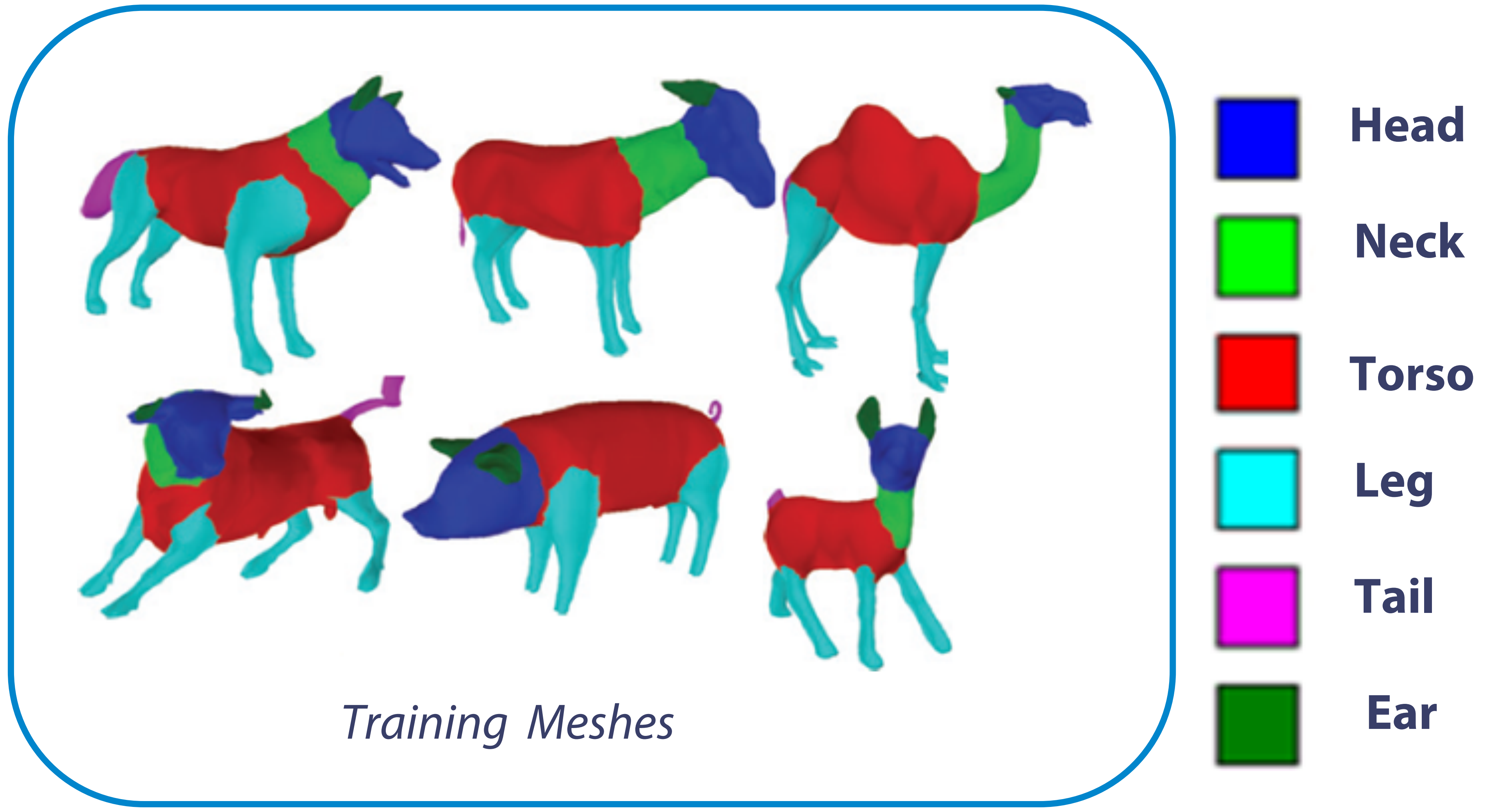
Supervised methods

Supervised mesh segmentation

[Kalogerakis et al 10]



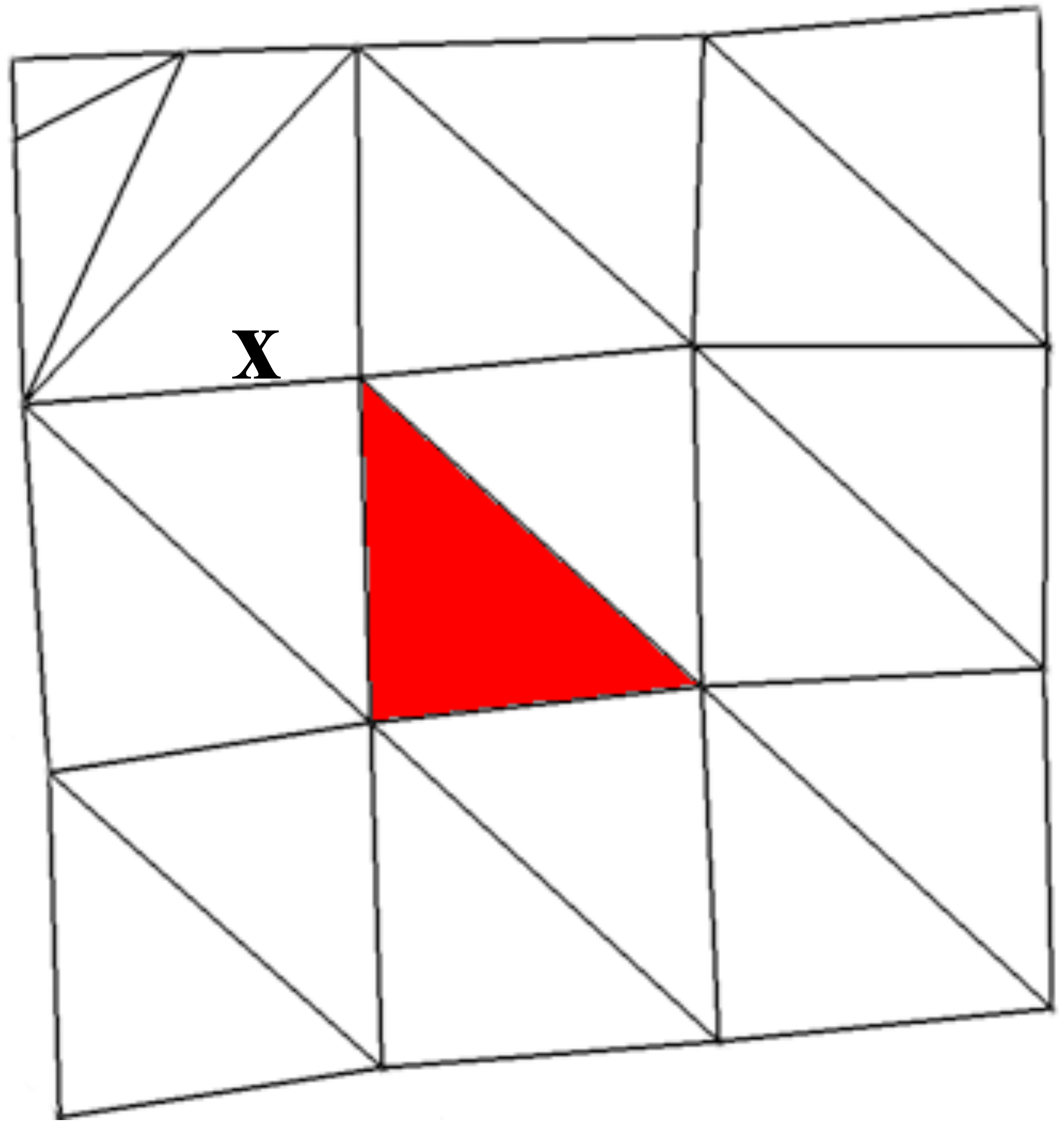
Find feature descriptors that characterize each label



Training Meshes

What feature descriptors?

Candidates: dimension 375 [Kalogerakis et al 10]



surface curvature

singular values from PCA

shape diameter

distances from medial surface

average geodesic distances

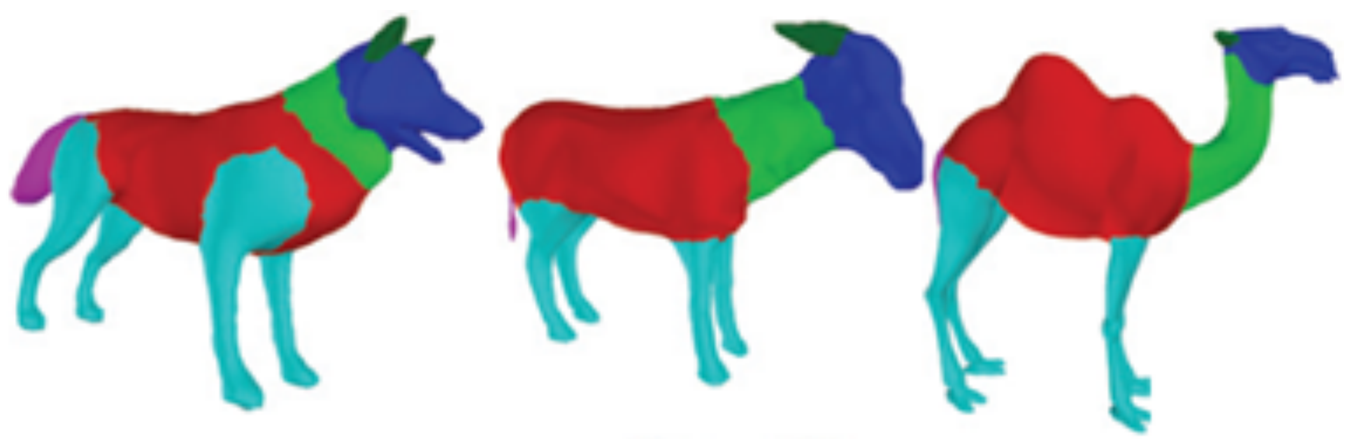
shape contexts

spin images

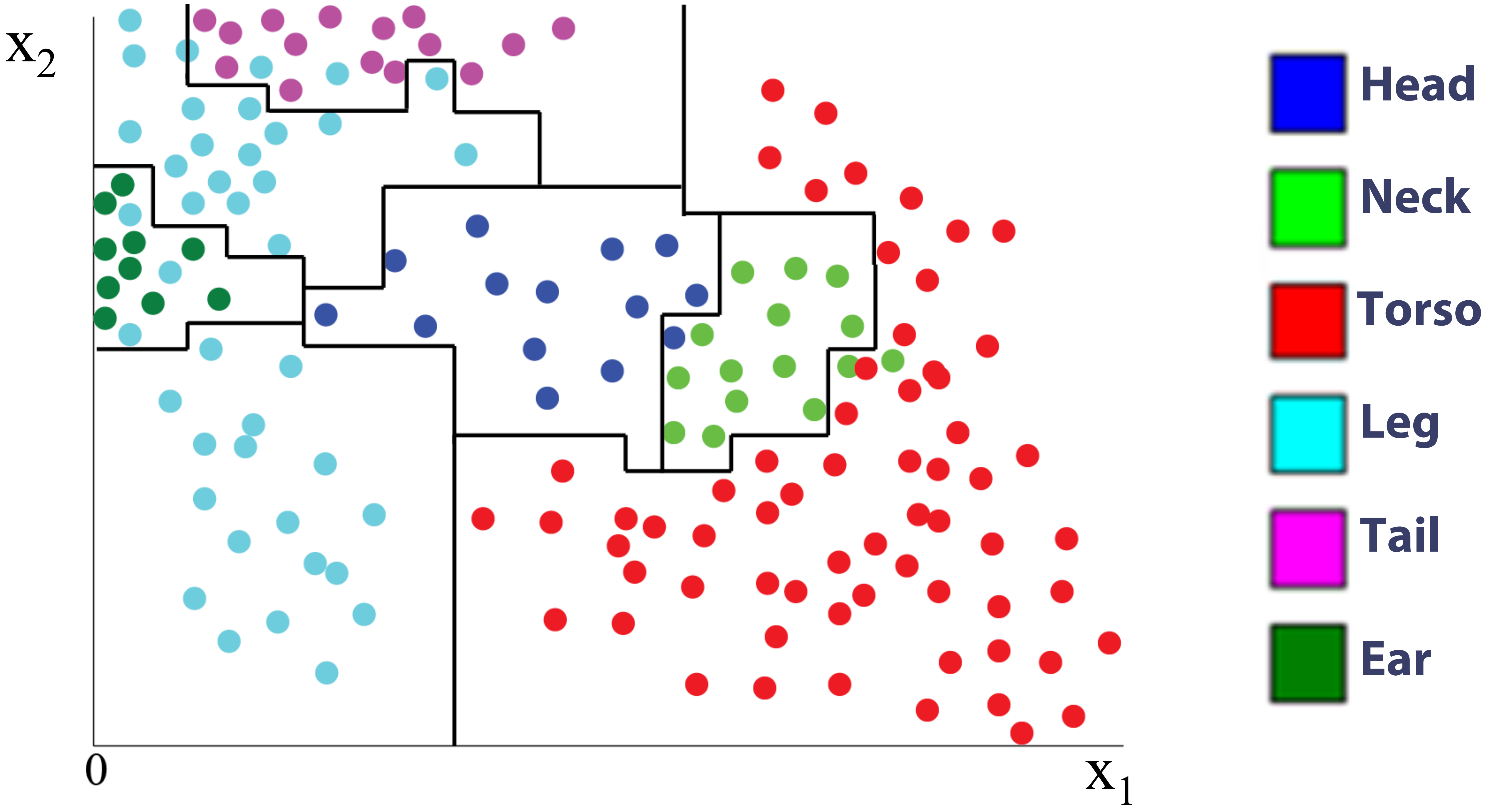
contextual label features

Learning a classifier

[Kalogerakis et al 10]



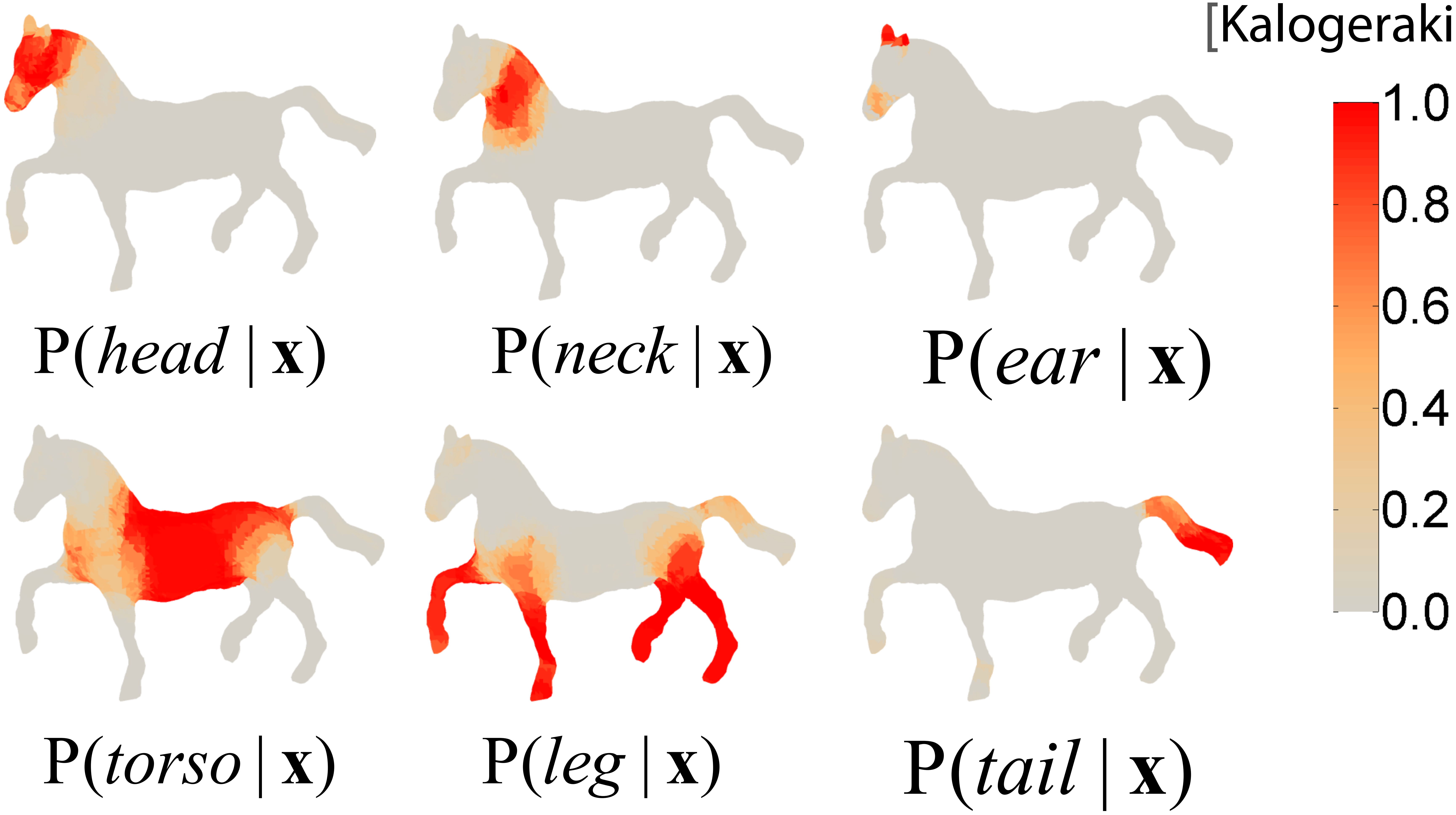
$$\rightarrow \{(\mathbf{x}_i, c_i)\}$$



Jointboost classifier **[Torralba et al. 2007]**

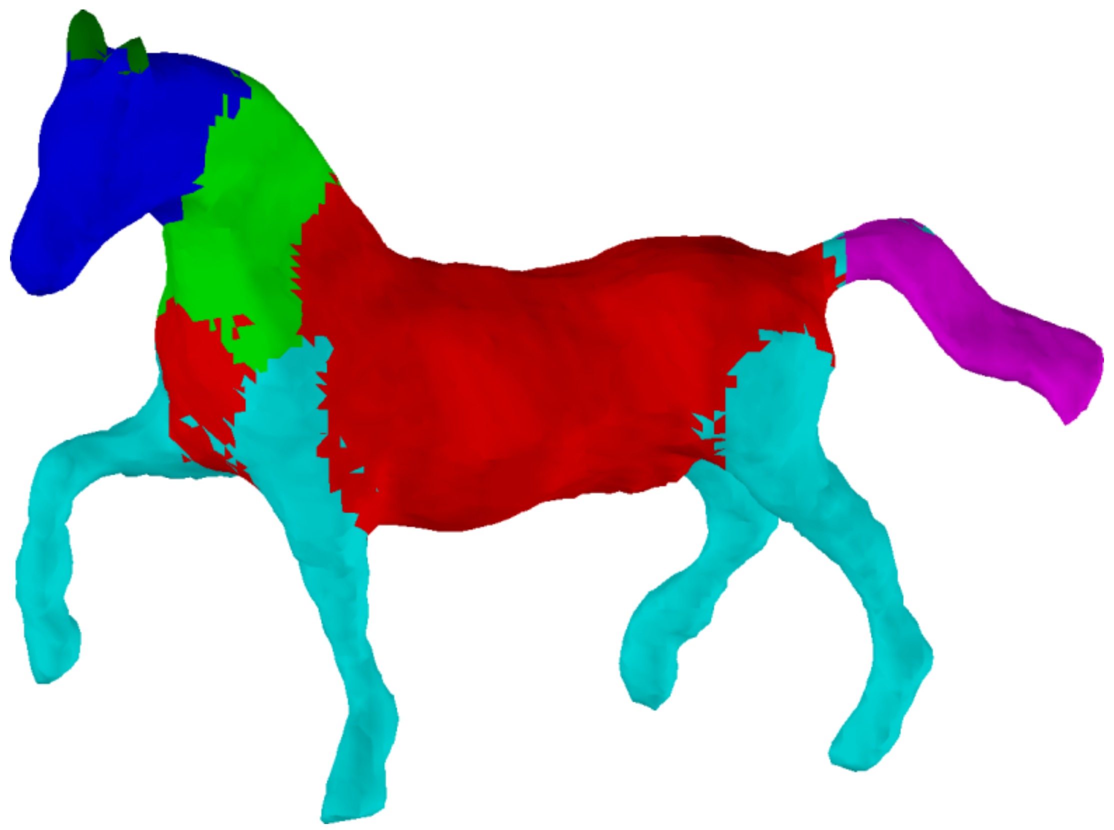
Inference

[Kalogerakis et al 10]



Inference

[Kalogerakis et al 10]



Most-likely labels

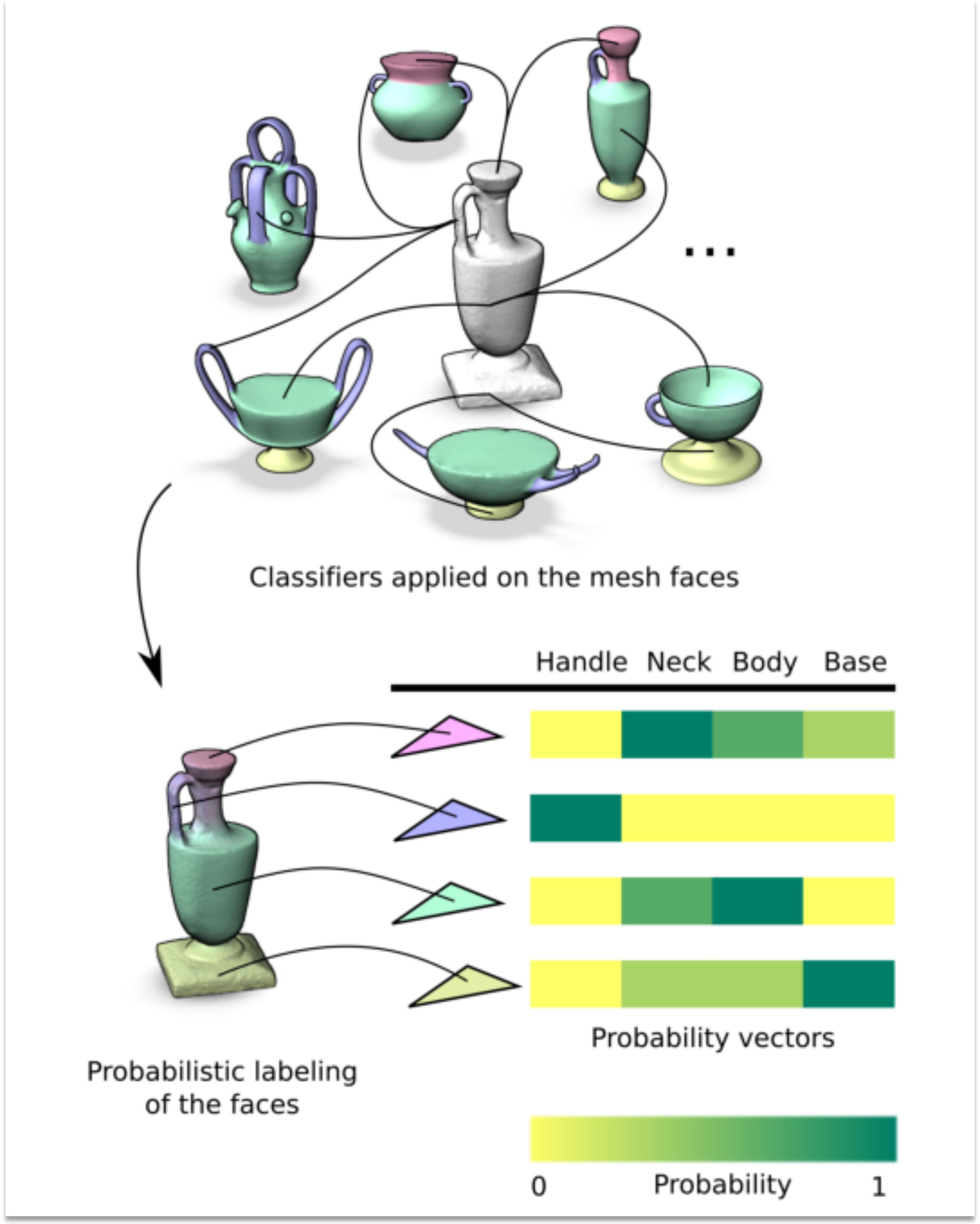
Final result

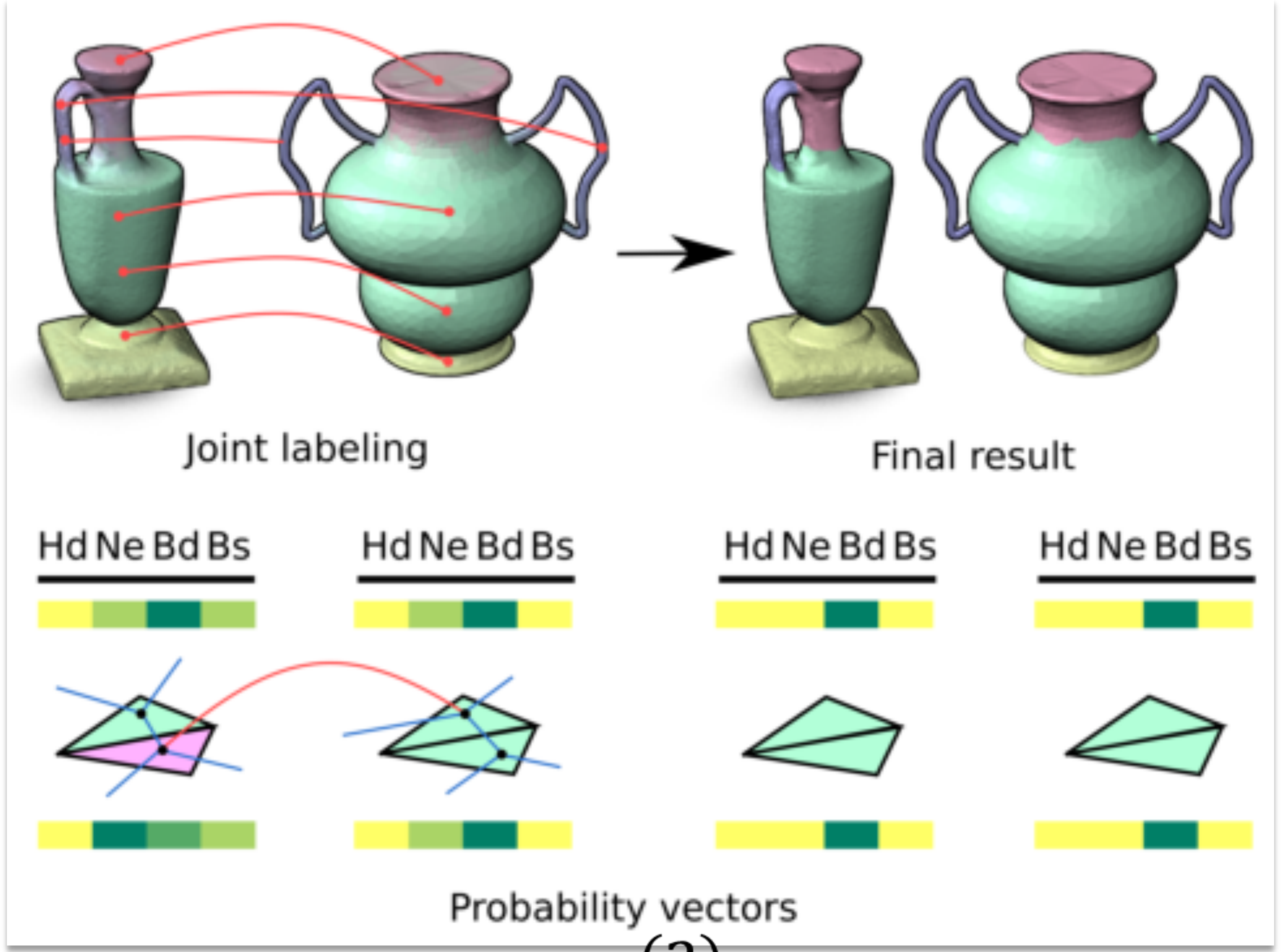
- Head
- Neck
- Torso
- Leg
- Tail
- Ear

$$c^* = \arg\min_c \sum_i -\log(P(c_i|x_i)) + \sum_{ij} E_2(c_i, c_j, y_{ij})$$

Pair-wise term

Problem modeling is important





$p^{(2)}$ – **pairwise**

All pair-wise terms

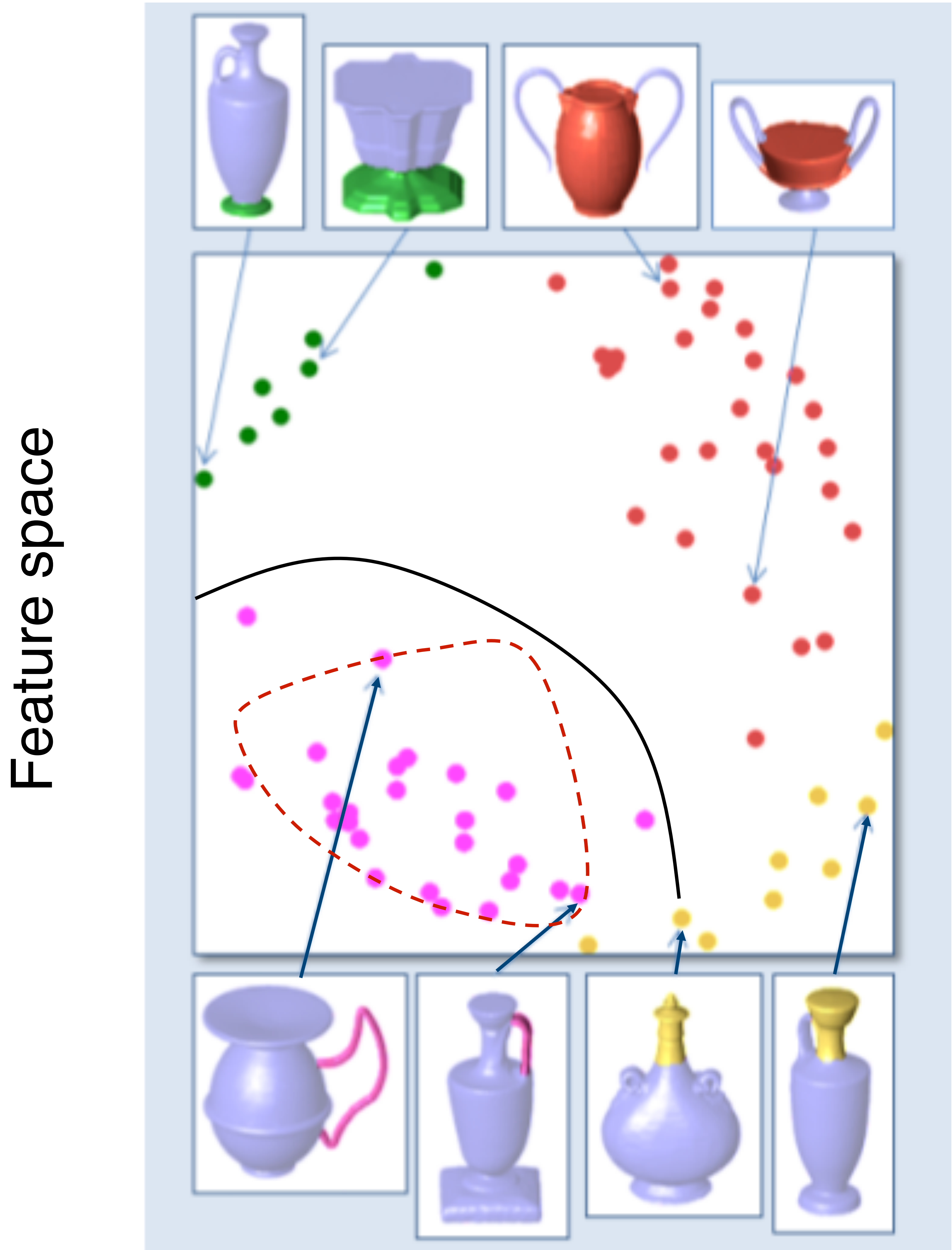
$p^{(1)}$ – **unary**

[van Kaik et al. EG 2011]

Supervised methods

Embedded spaces

[Sidi et al 11]



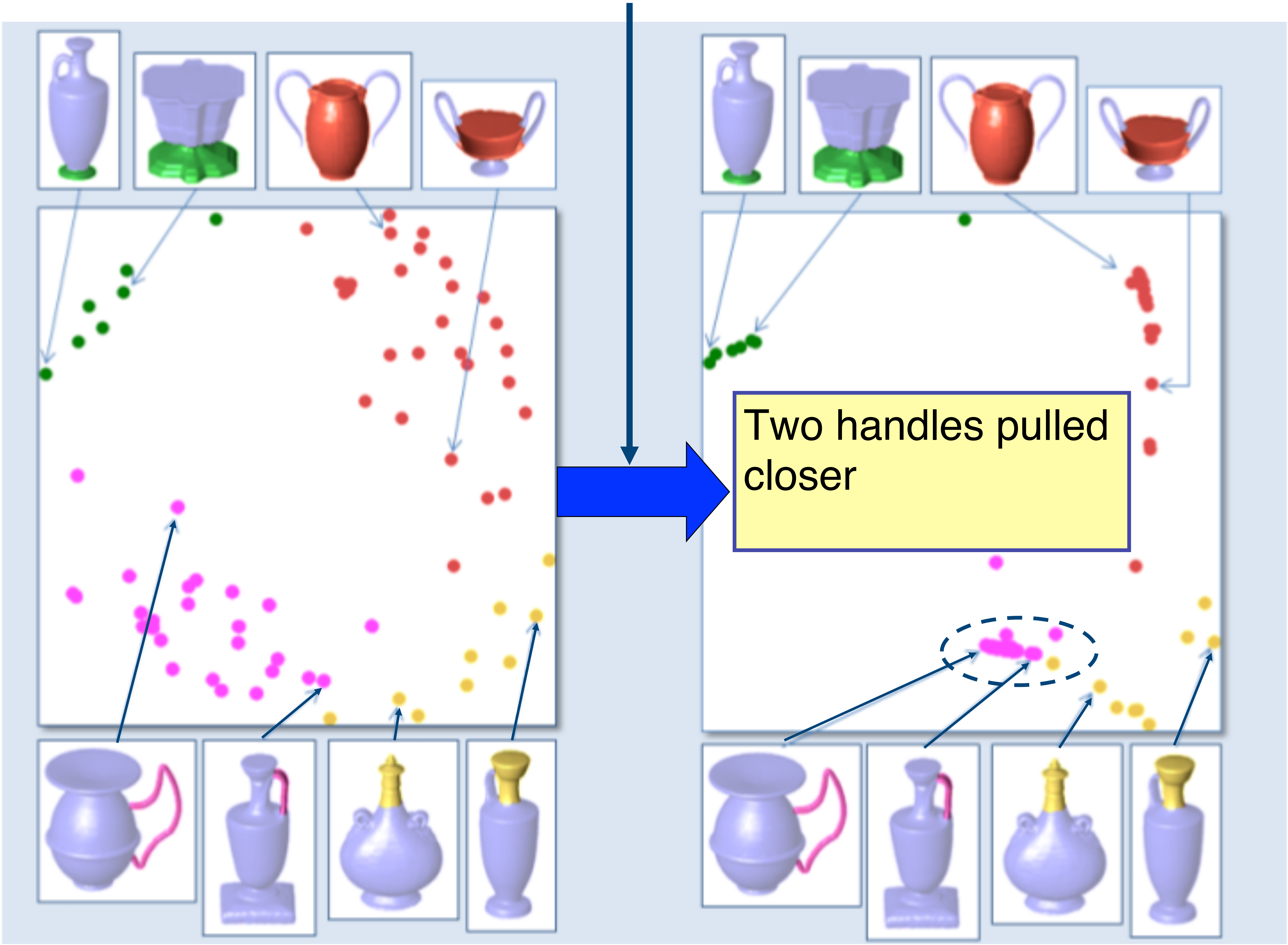
Shape segments mapped to some feature space

Segments of the same class form a (graph) cluster

After a “spectral transform”

[Sidi et al 11]

Feature space

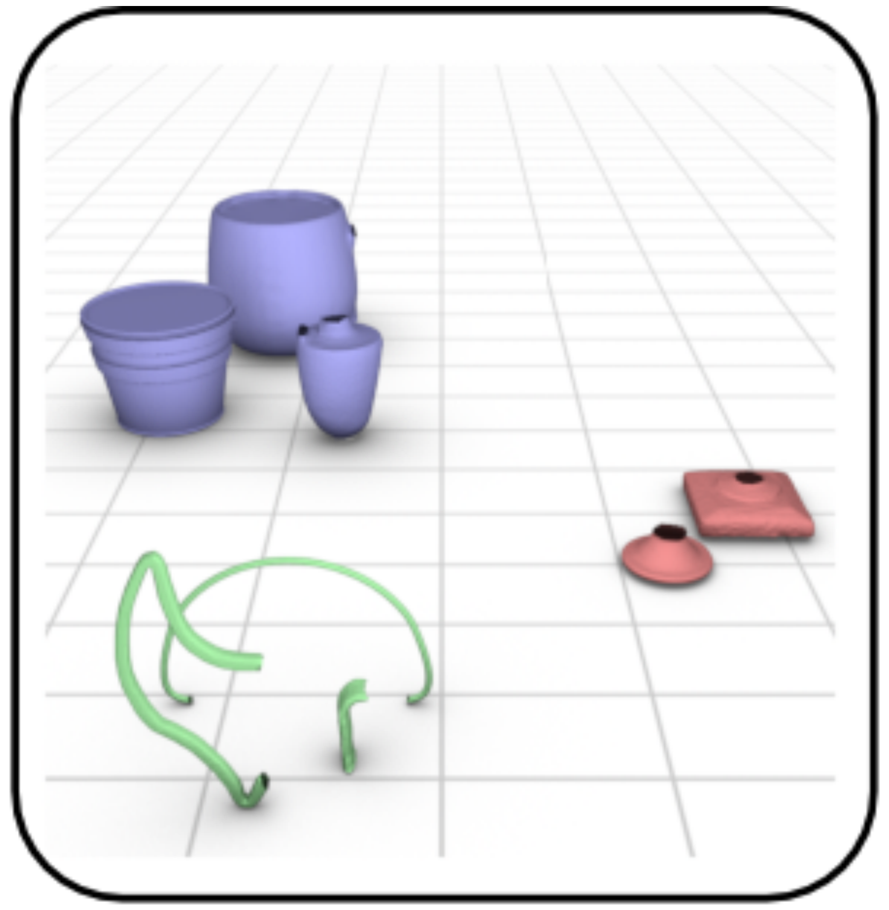


Algorithm

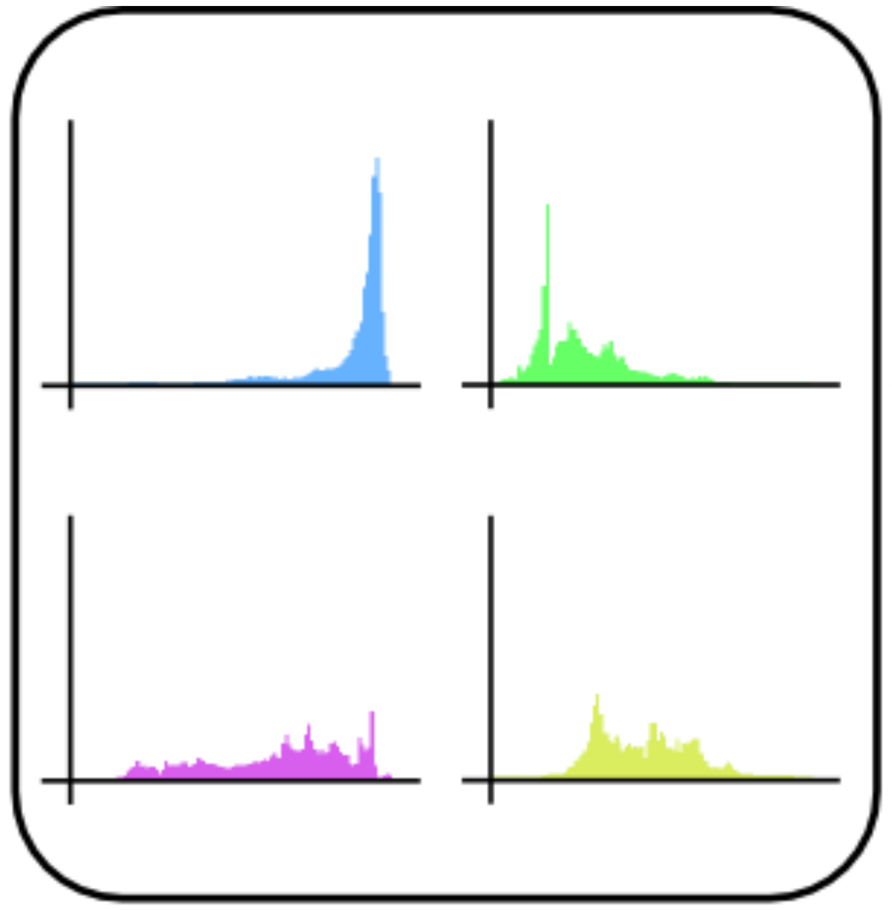
[Sidi et al 11]

(a) Per-object segmentation

(b) Diffusion maps



(c) Clustering

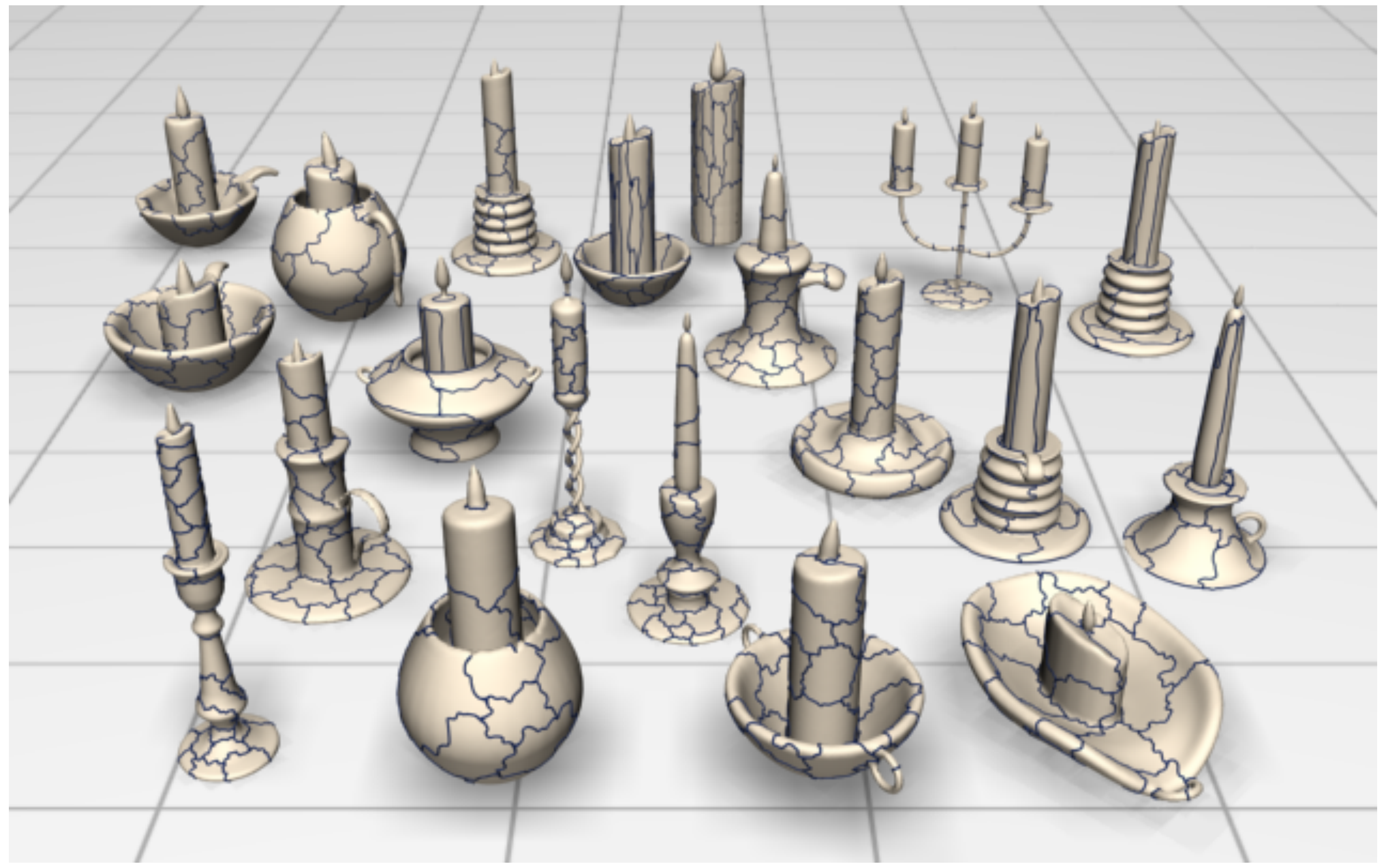


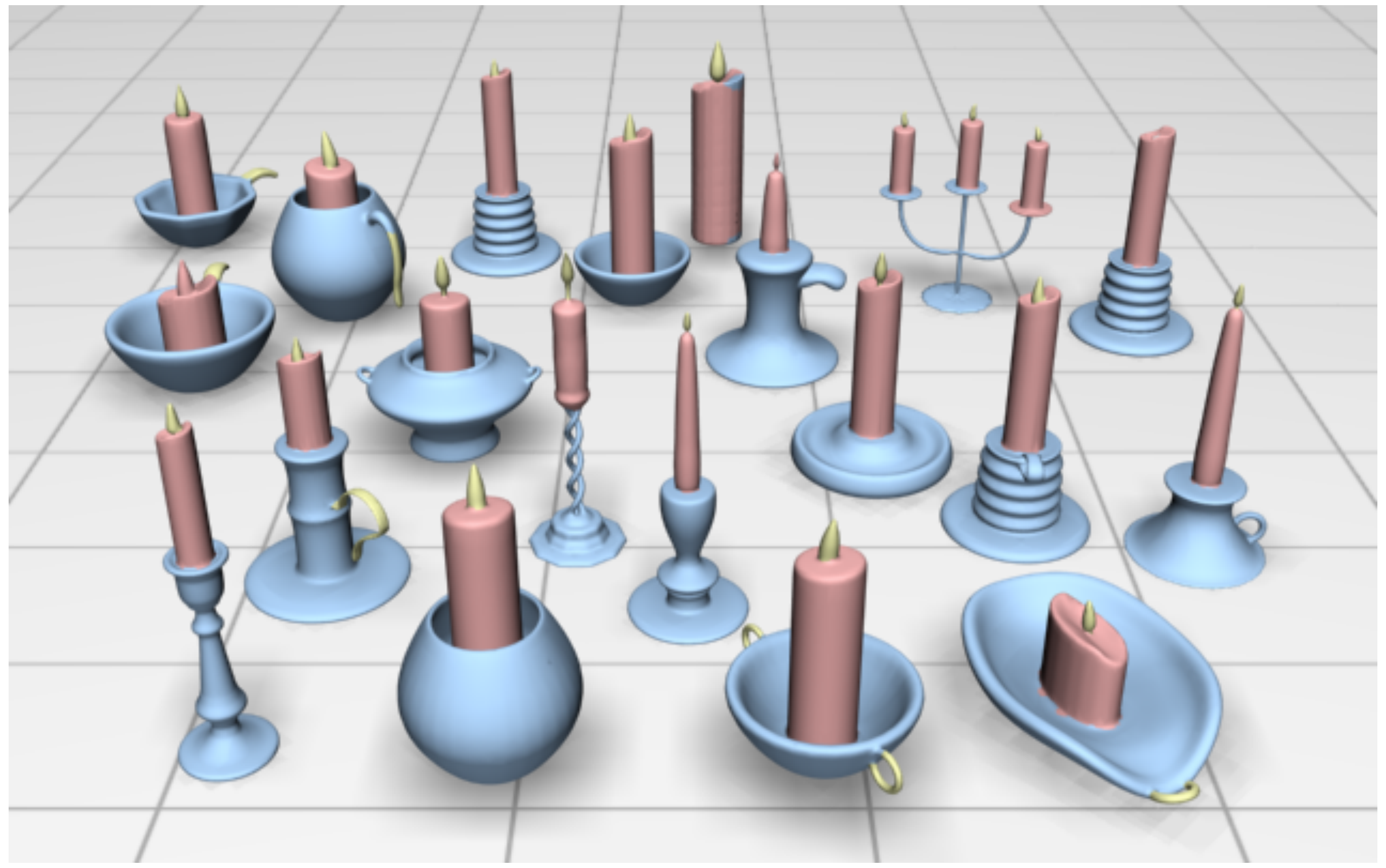
(d) Statistical model

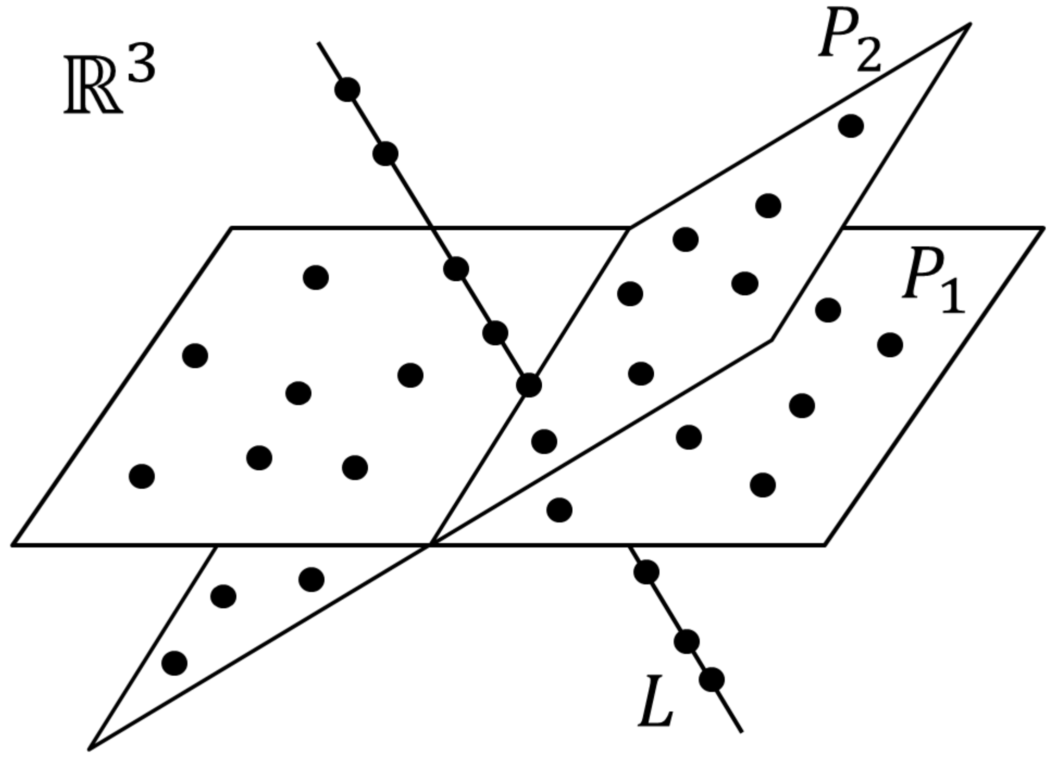
(e) Result

Sub-space clustering

[Hu et al 12]

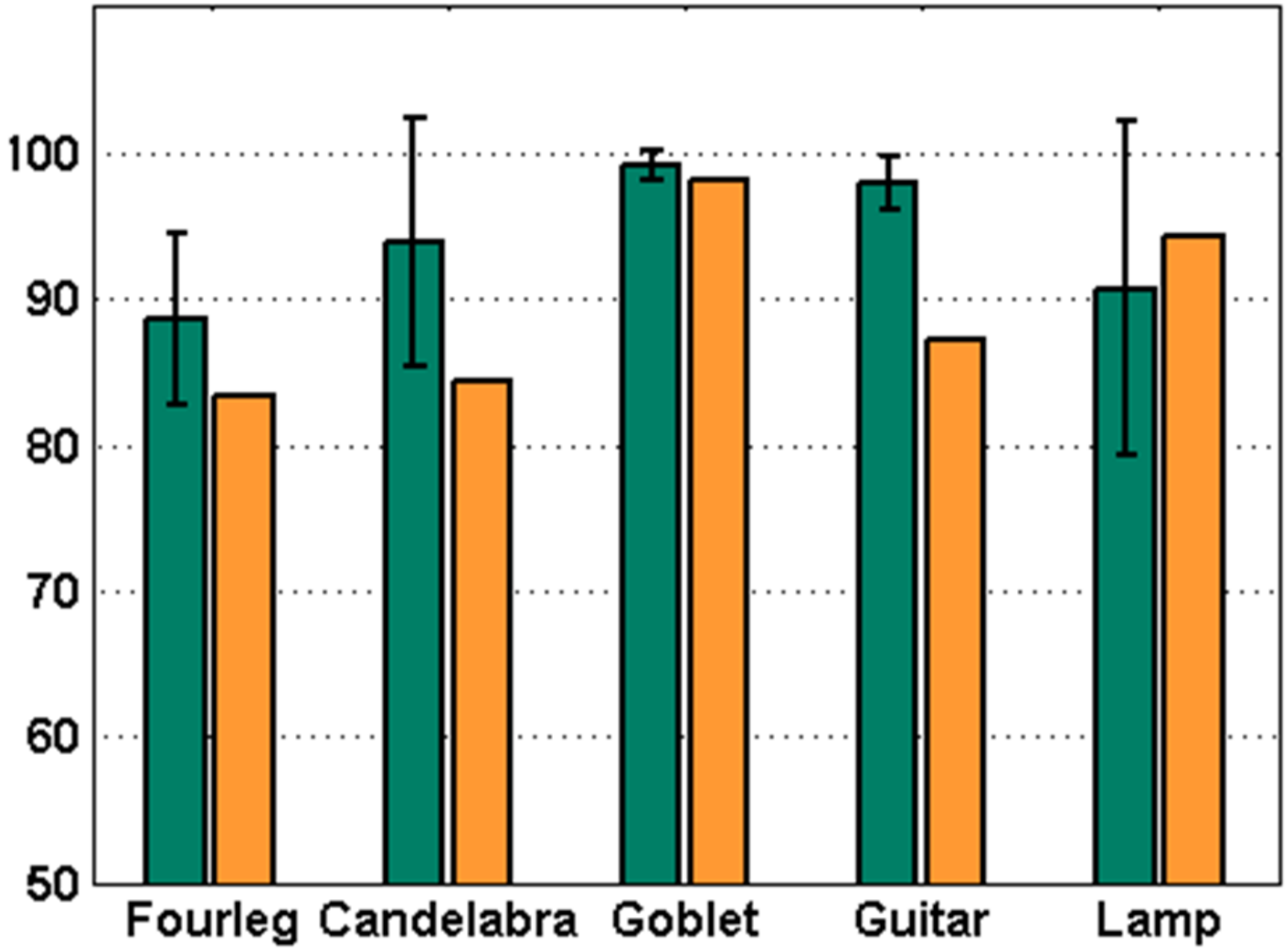






Features inspired from supervised learning [Kalogerakis et al 10]

Comparison – rand index



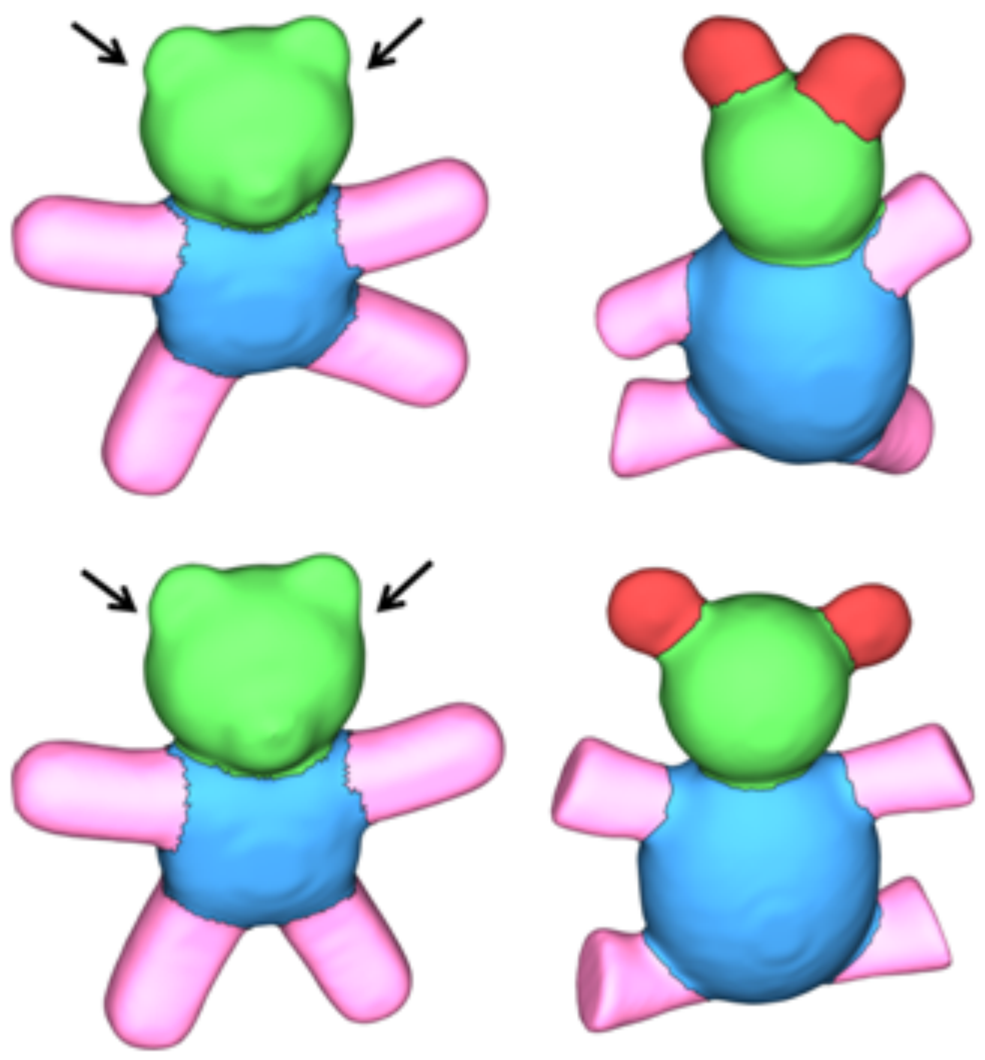
Joint shape segmentation

[Huang et al 11]

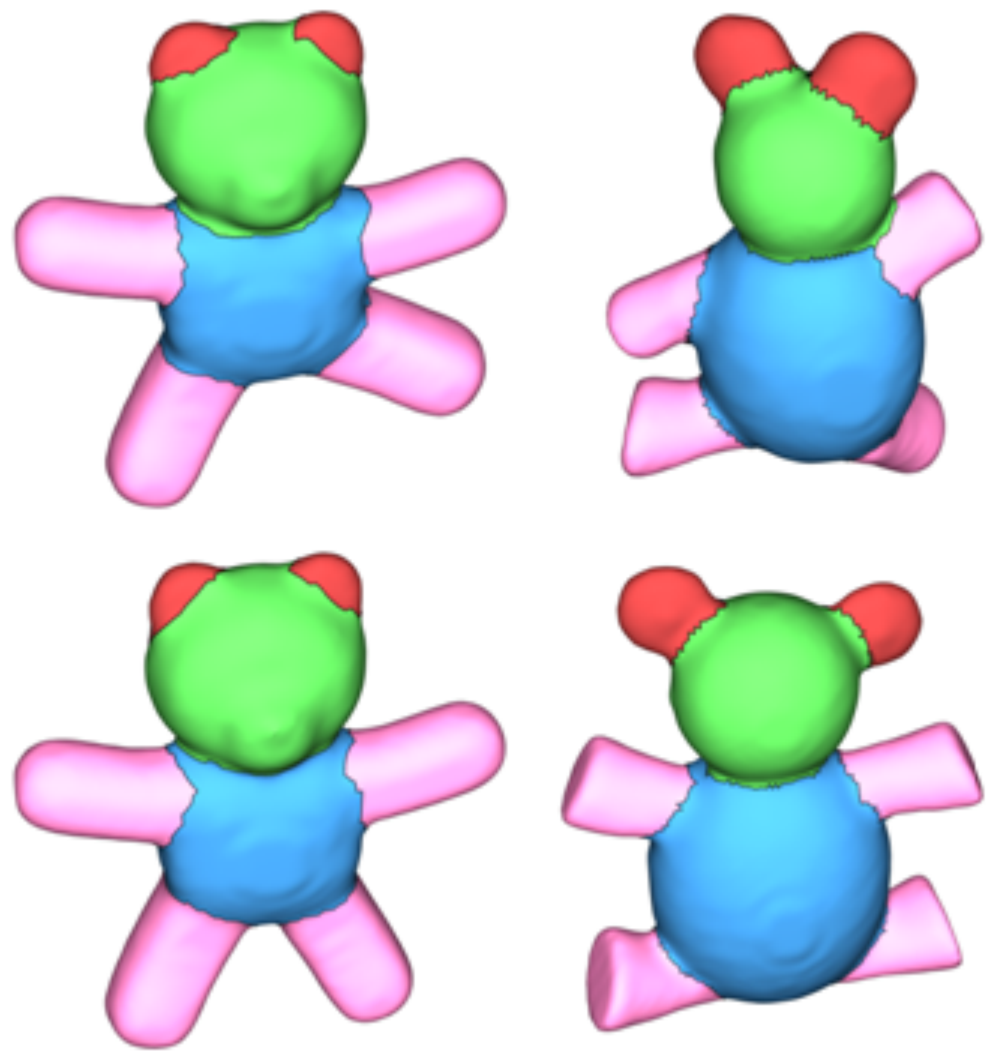
Structural similarity of segmentations

- Low saliency

Single shape segmentation



Co-segmentation



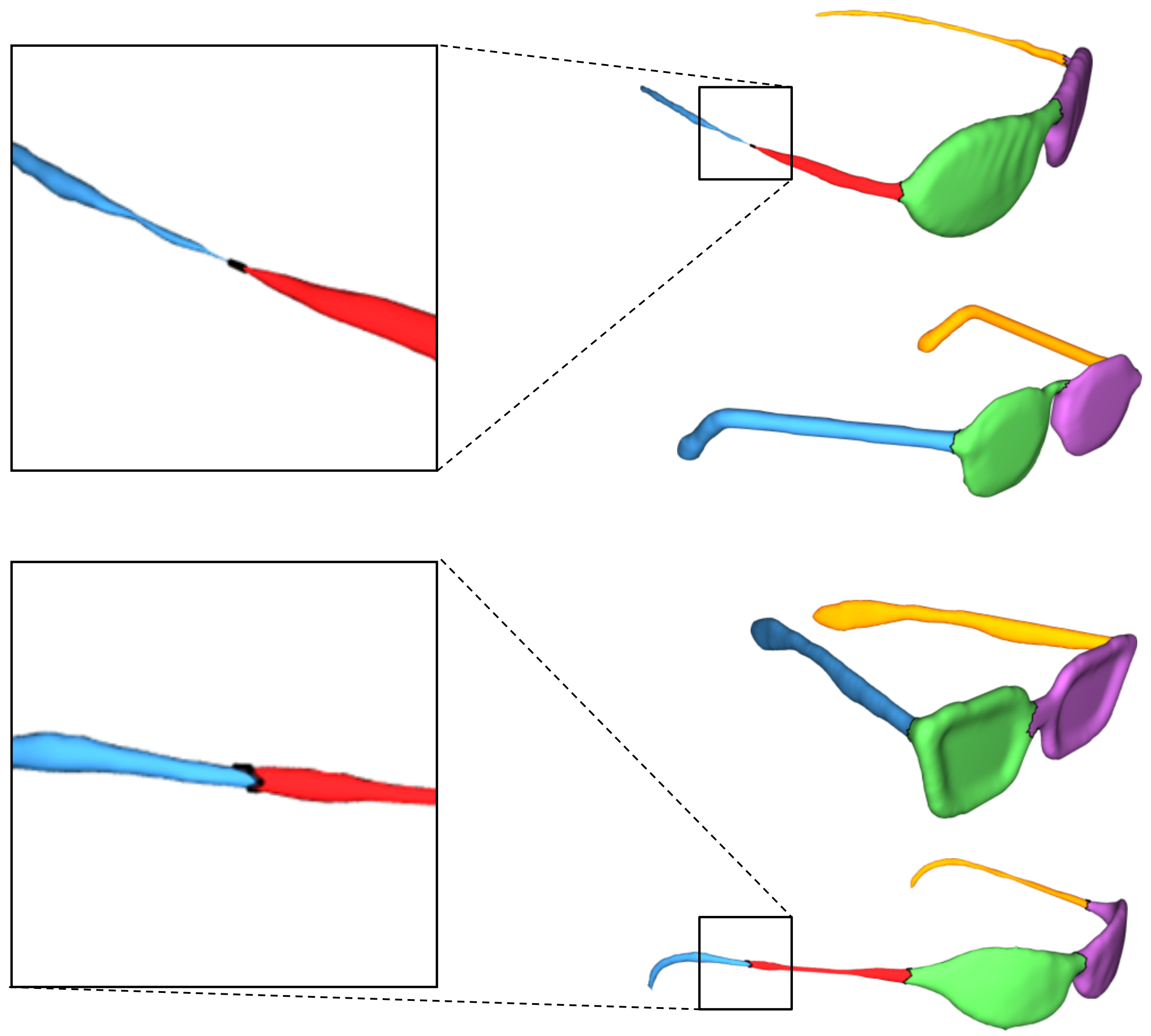
Joint shape segmentation

[Huang et al 11]

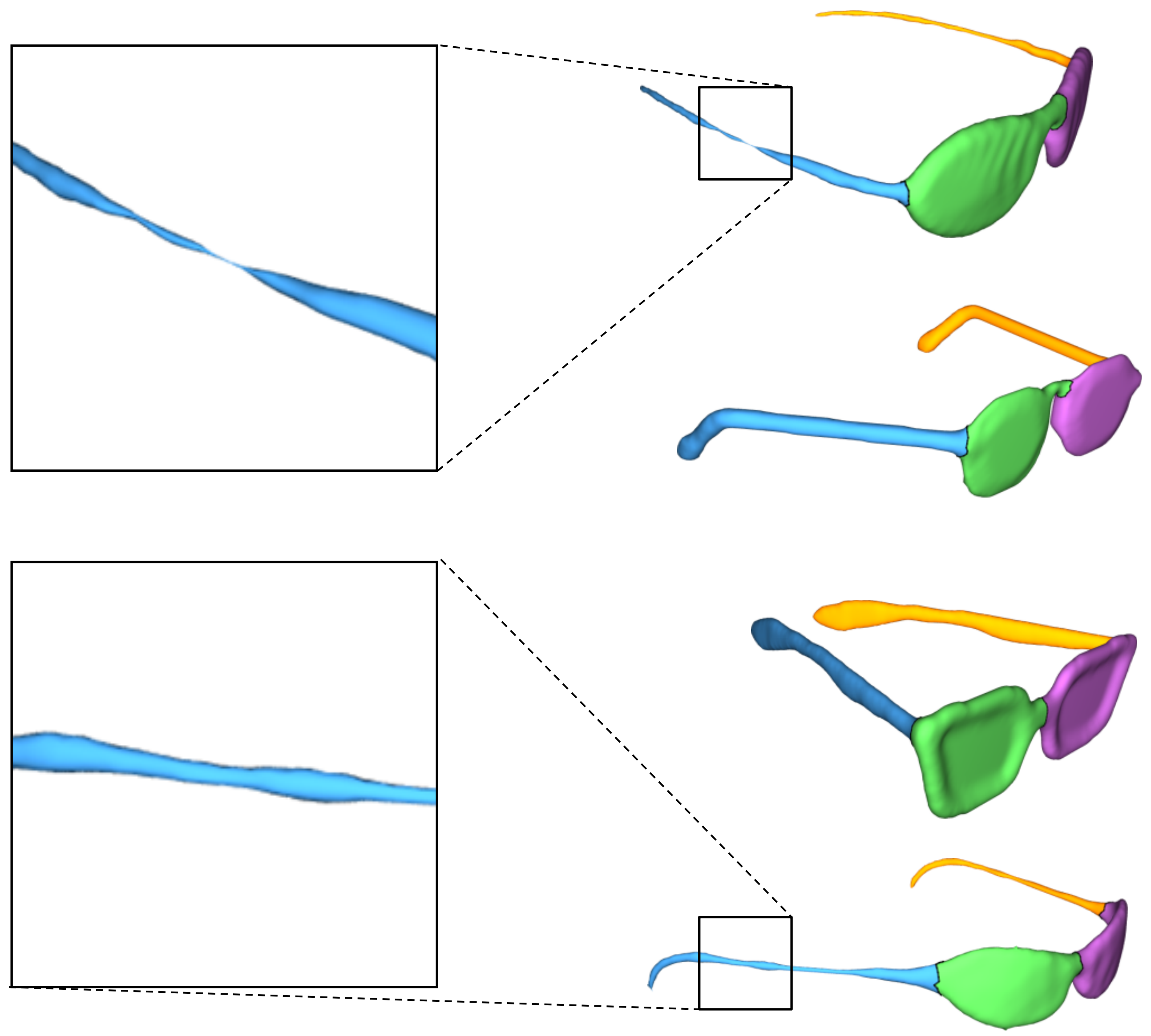
Structural similarity of segmentations

- Extraneous geometric clues

Single shape segmentation



Co-segmentation



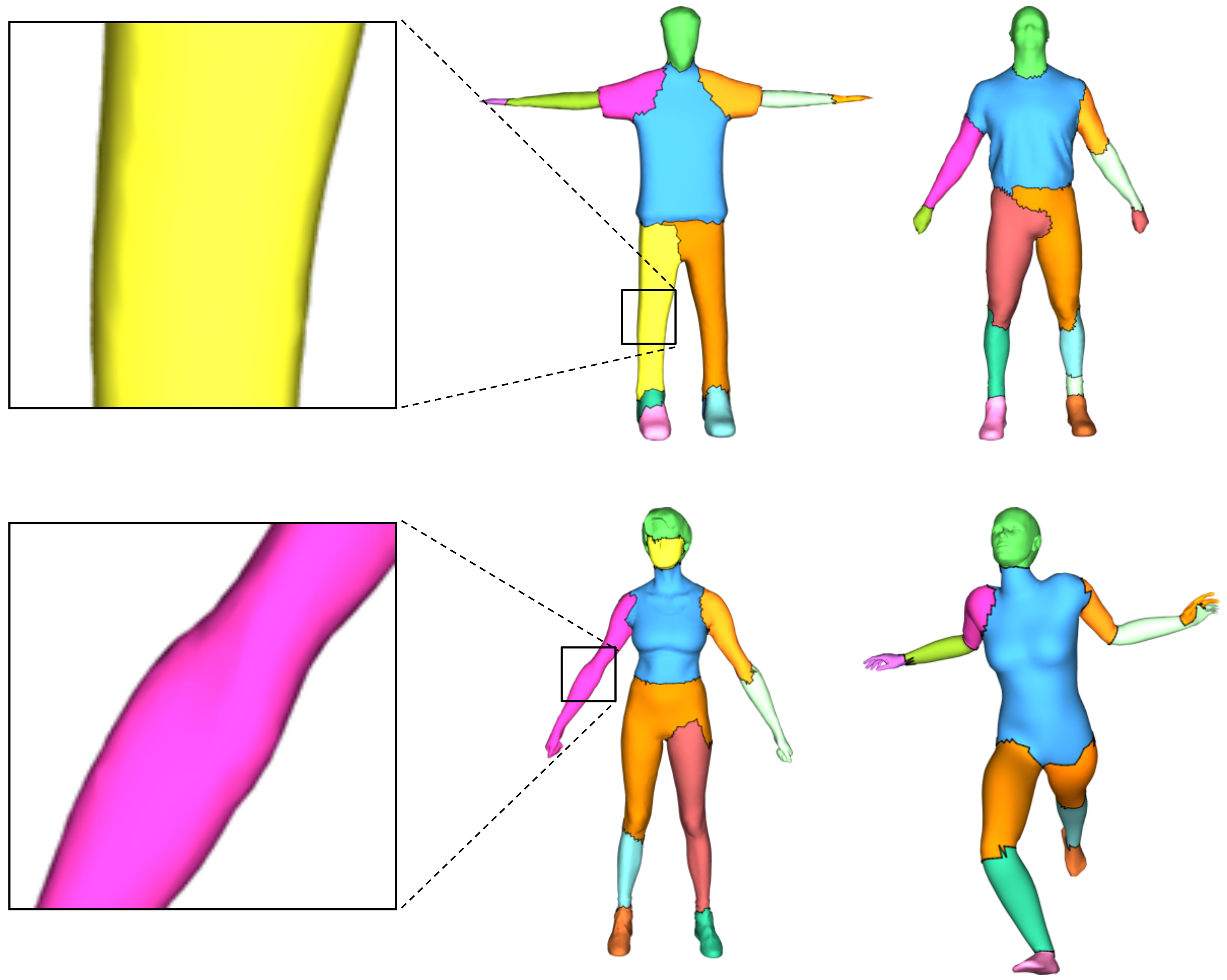
Joint shape segmentation

[Huang et al 11]

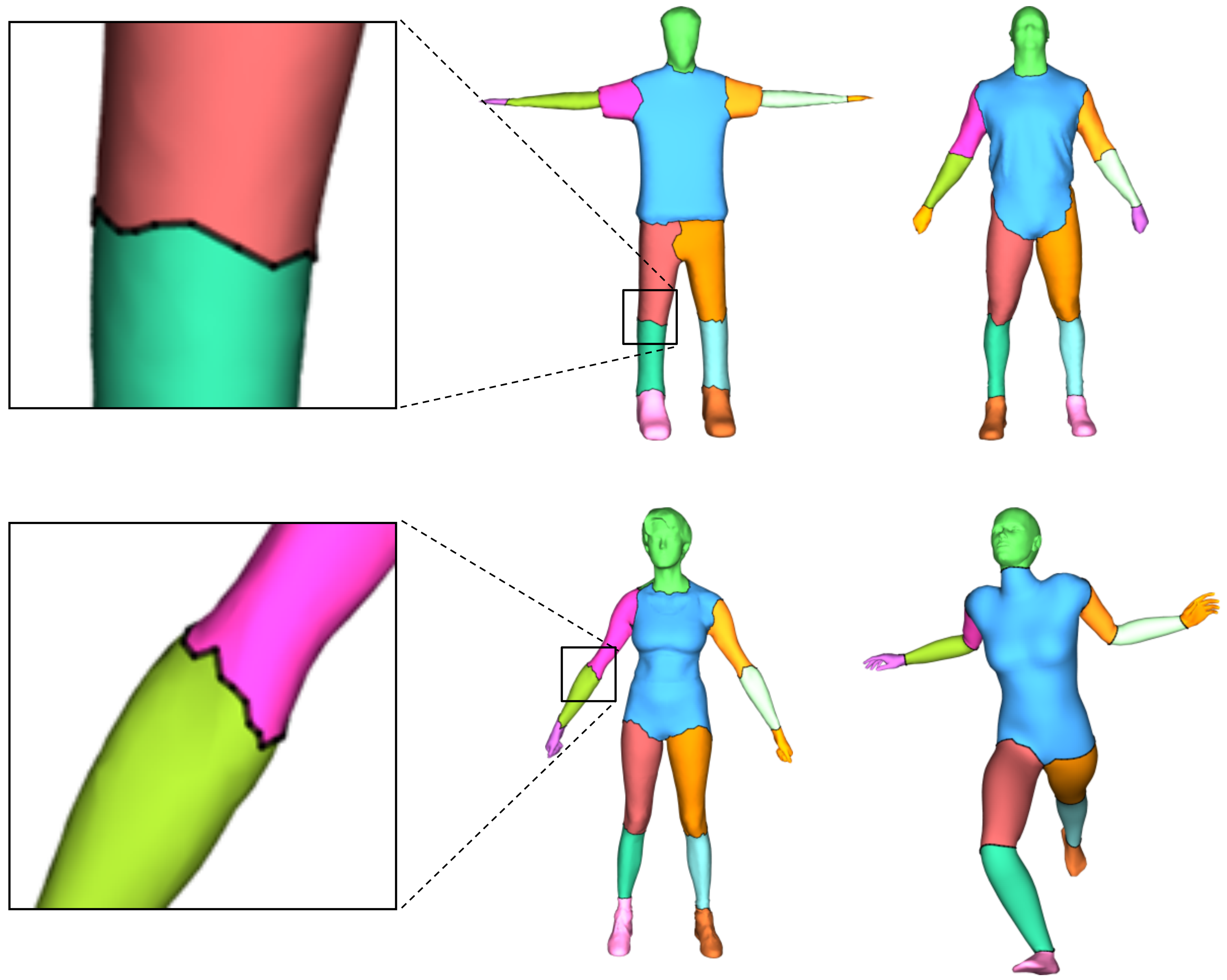
Invariance of segments

- Articulated structures

Single shape segmentation



Co-segmentation

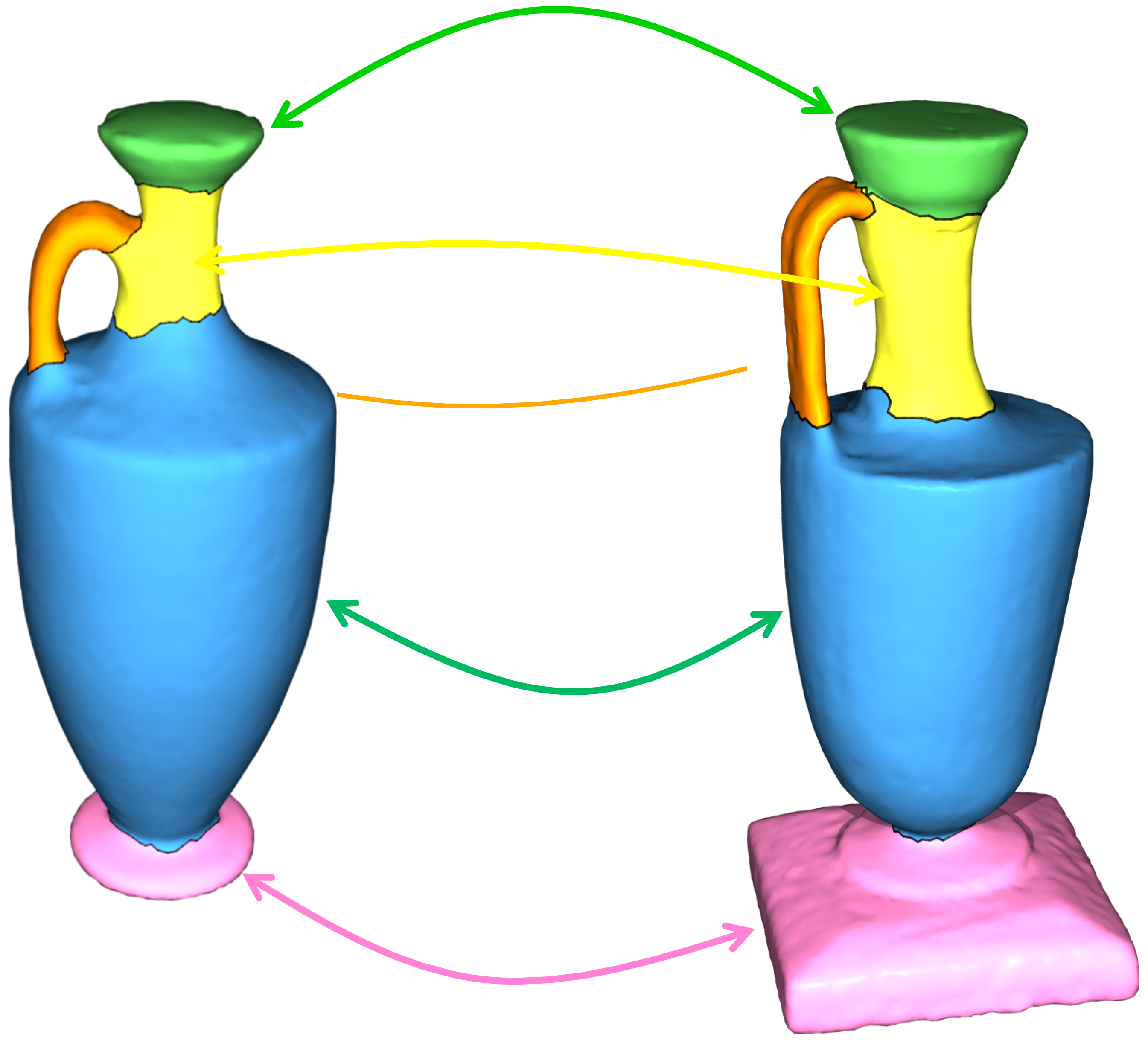


Joint shape segmentation

Objective:

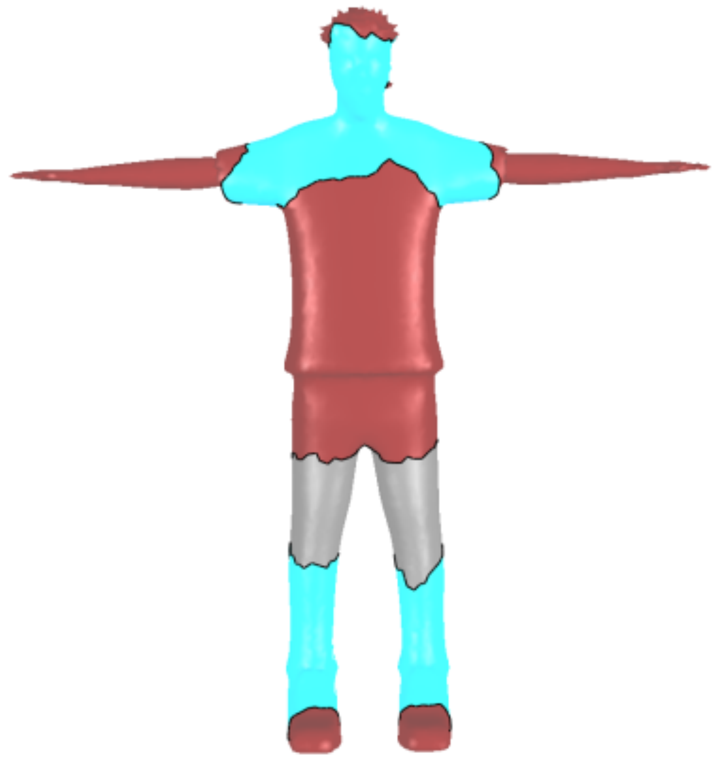
[Huang et al 11]

$$\max_{S_1, S_2} \text{score}(S_1) + \text{score}(S_2) + \text{consistency}(S_1, S_2)$$

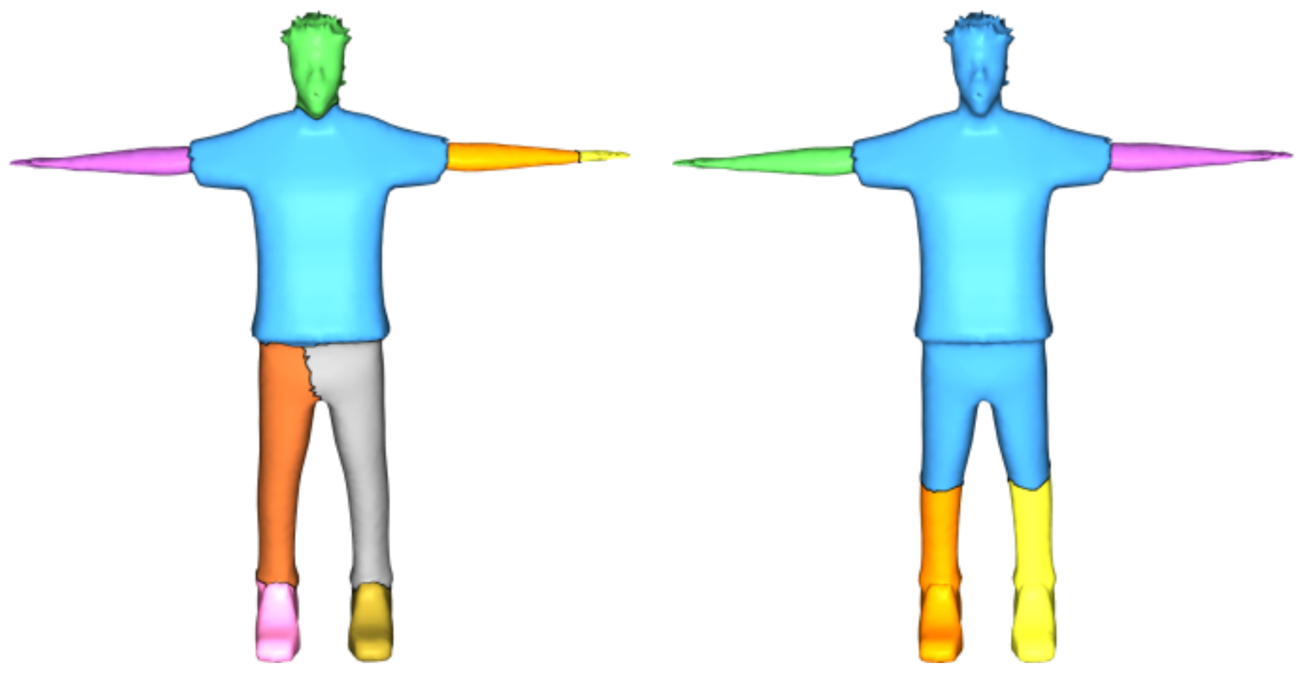


Segmentation parameterization

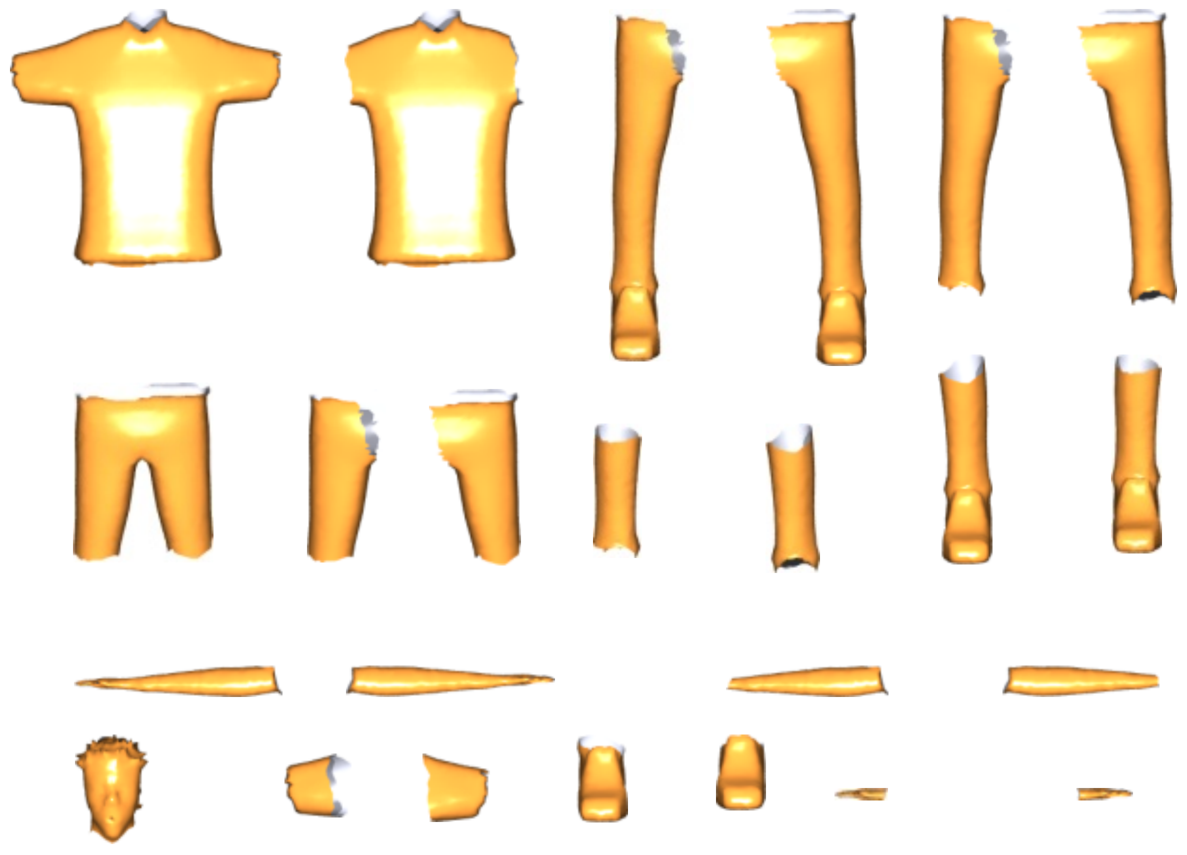
[Huang et al 11]



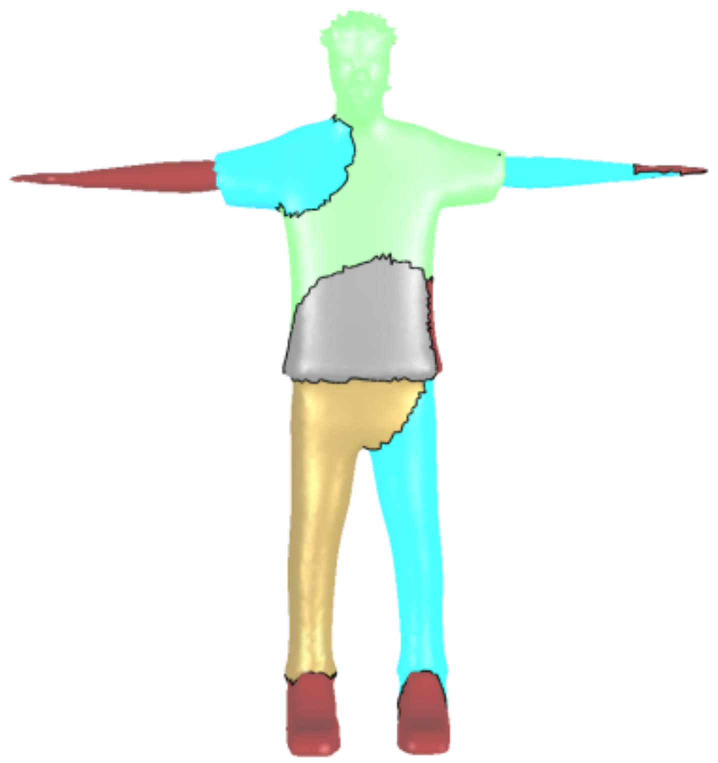
Shape Diameter
[Shapira et al. 08]



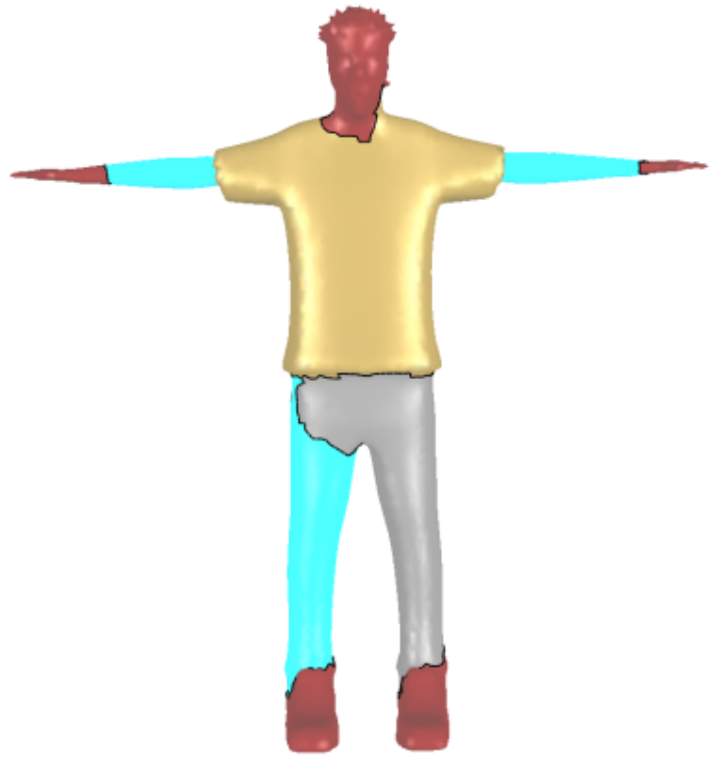
. . .
Randomized Cuts



. . .
Initial Segments



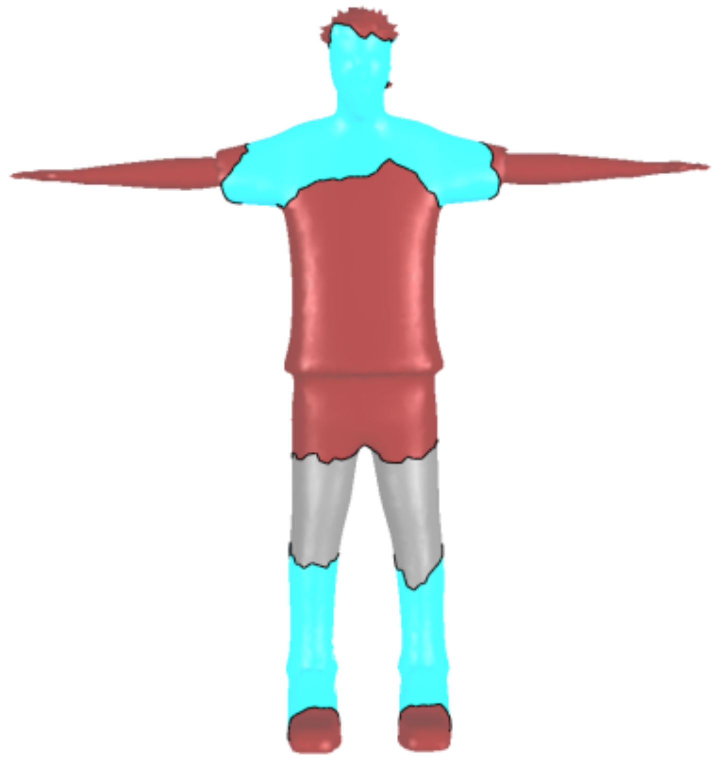
Random Walks
[Lai et al. 08]



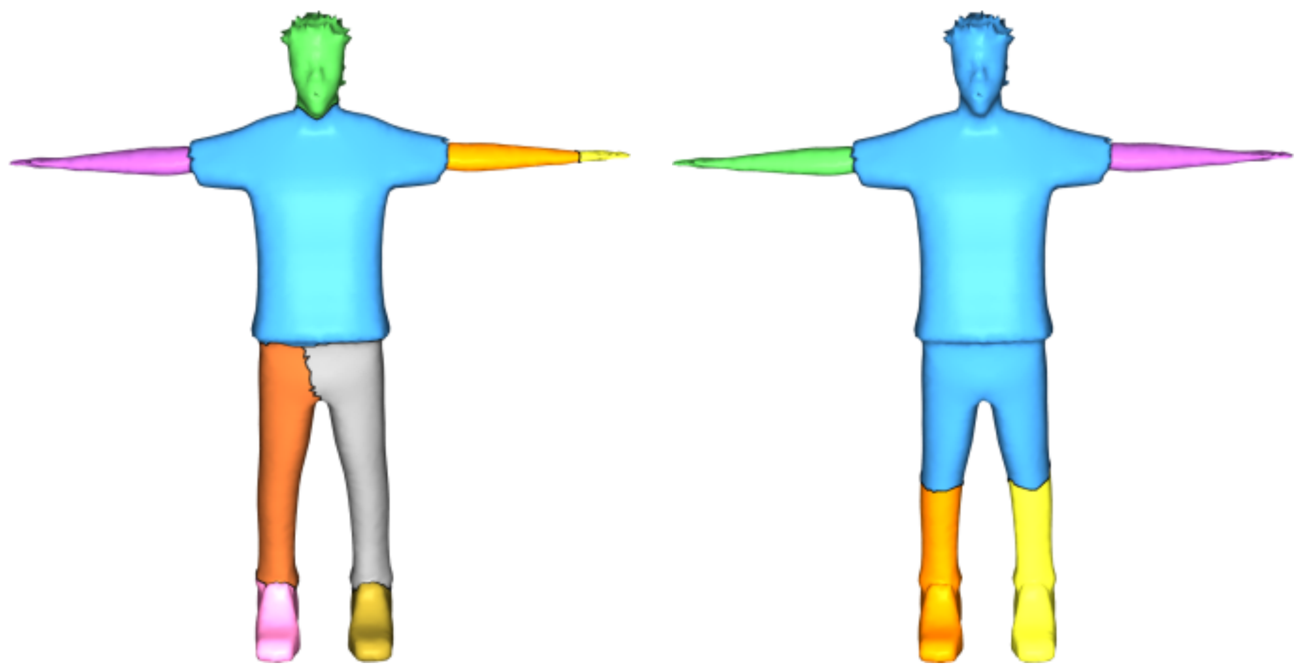
Normalized Cuts
[Golovinskiy and Funkhouser 08]

Segmentation parameterization

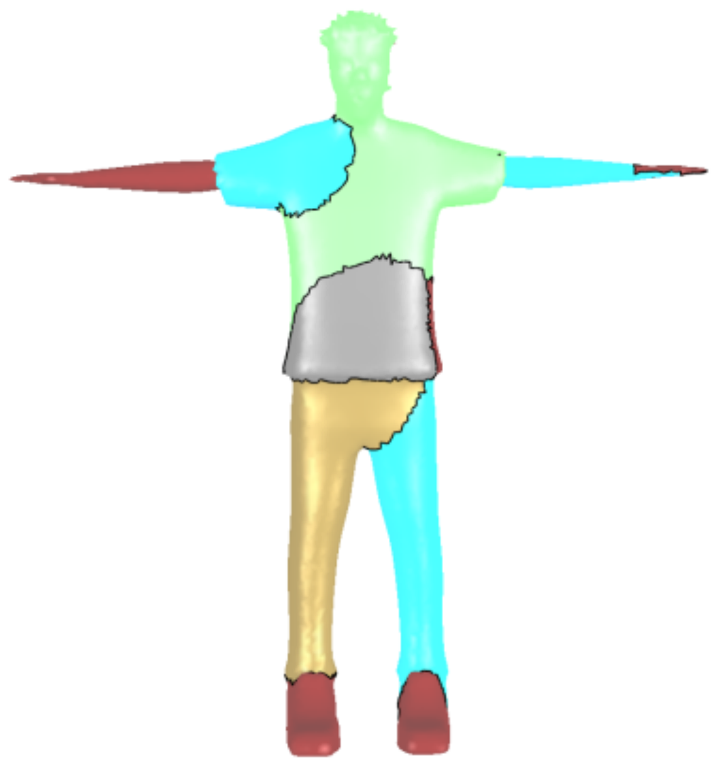
[Huang et al 11]



Shape Diameter
[Shapira et al. 08]



. . .
Randomized Cuts



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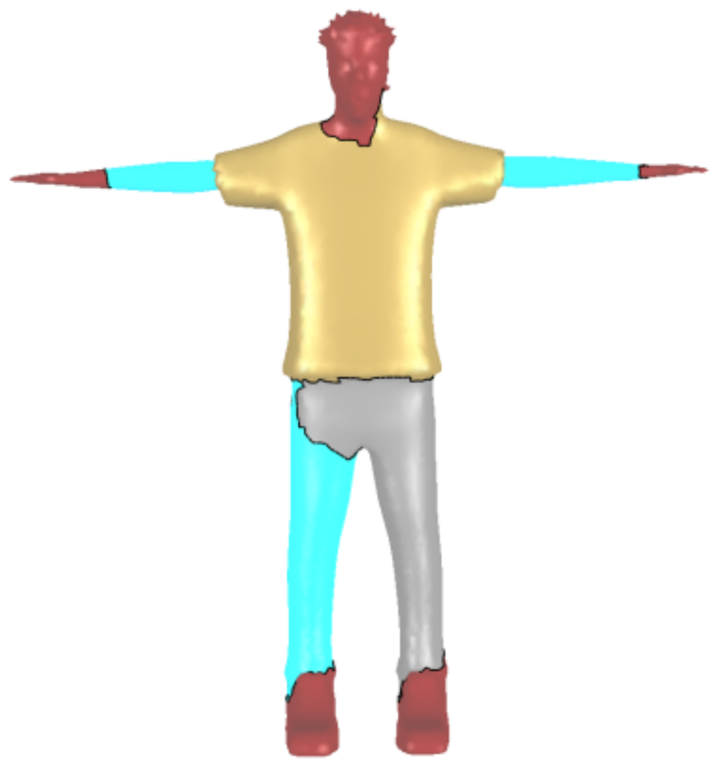

Normalized Cuts
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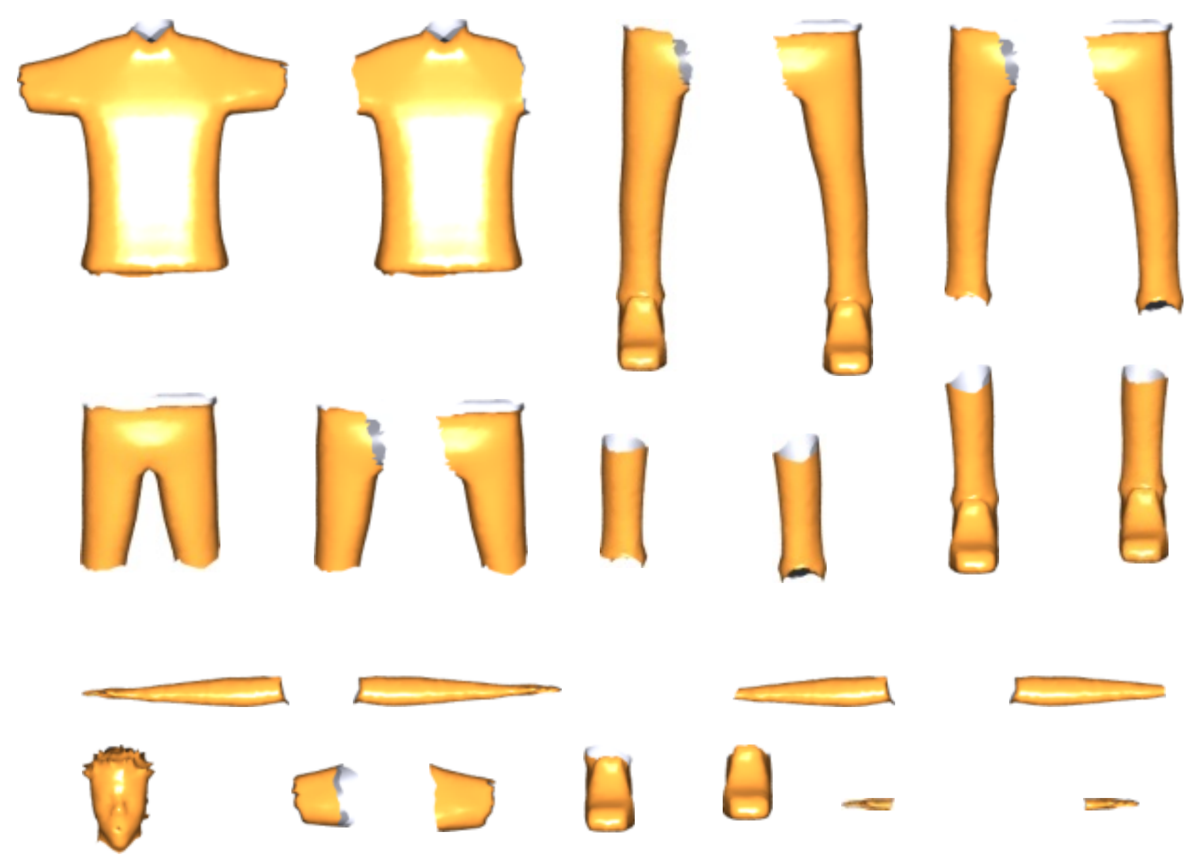
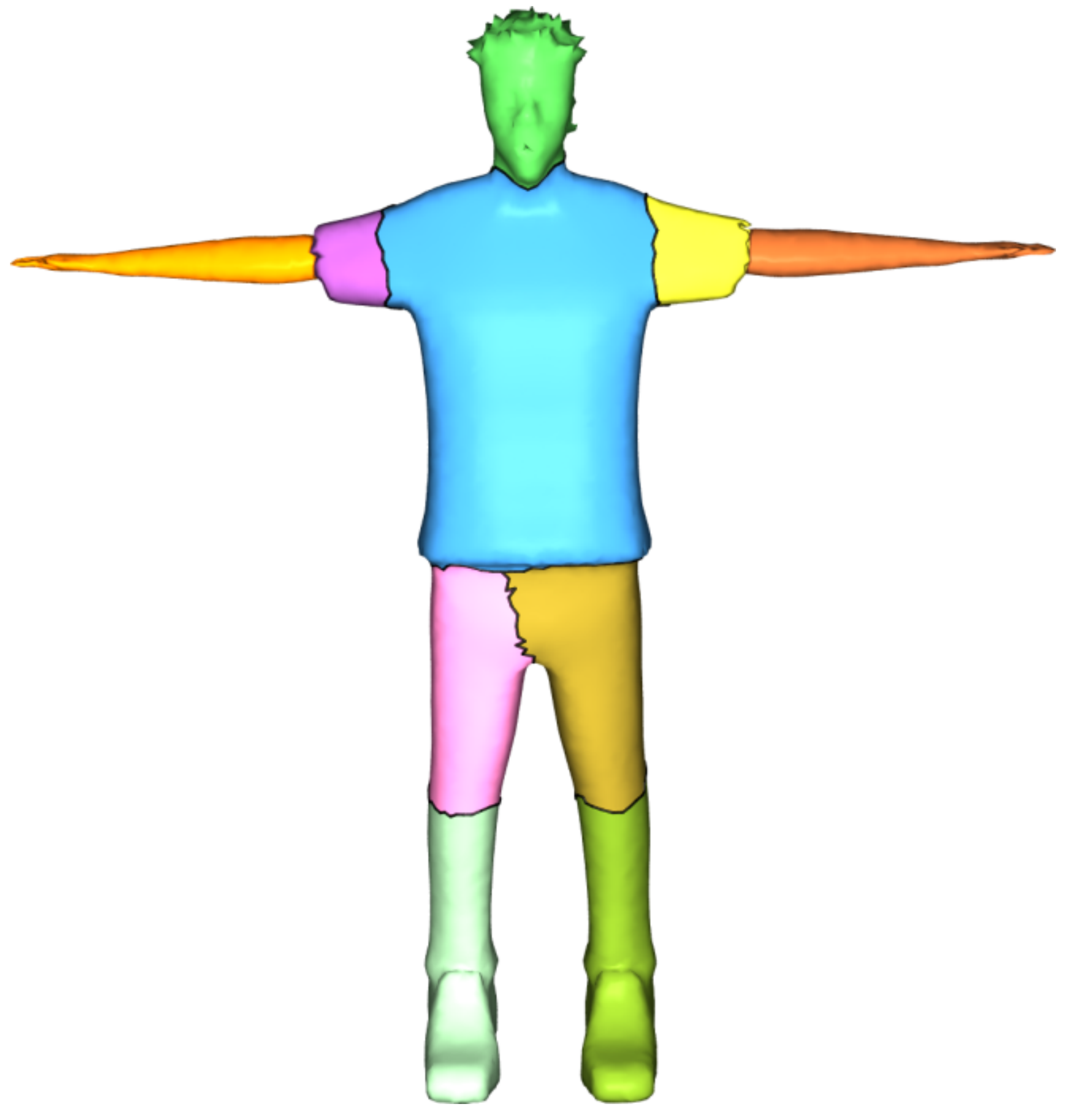


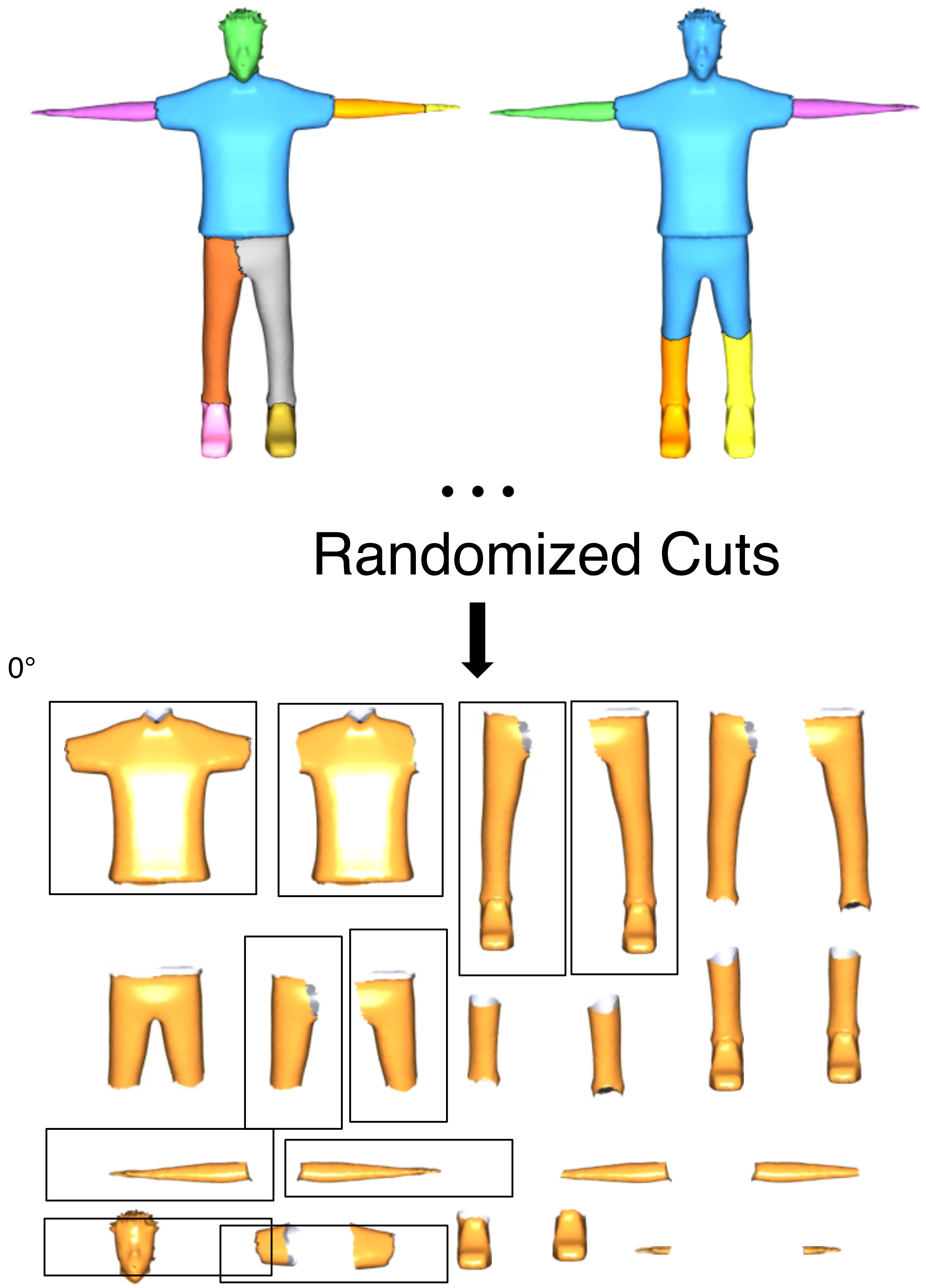
. . .
Initial Segments

Segmentation parameterization

Segmentations: subsets of initial segments obtained from randomized segmentations

[Huang et al 11]





Segmentation parameterization

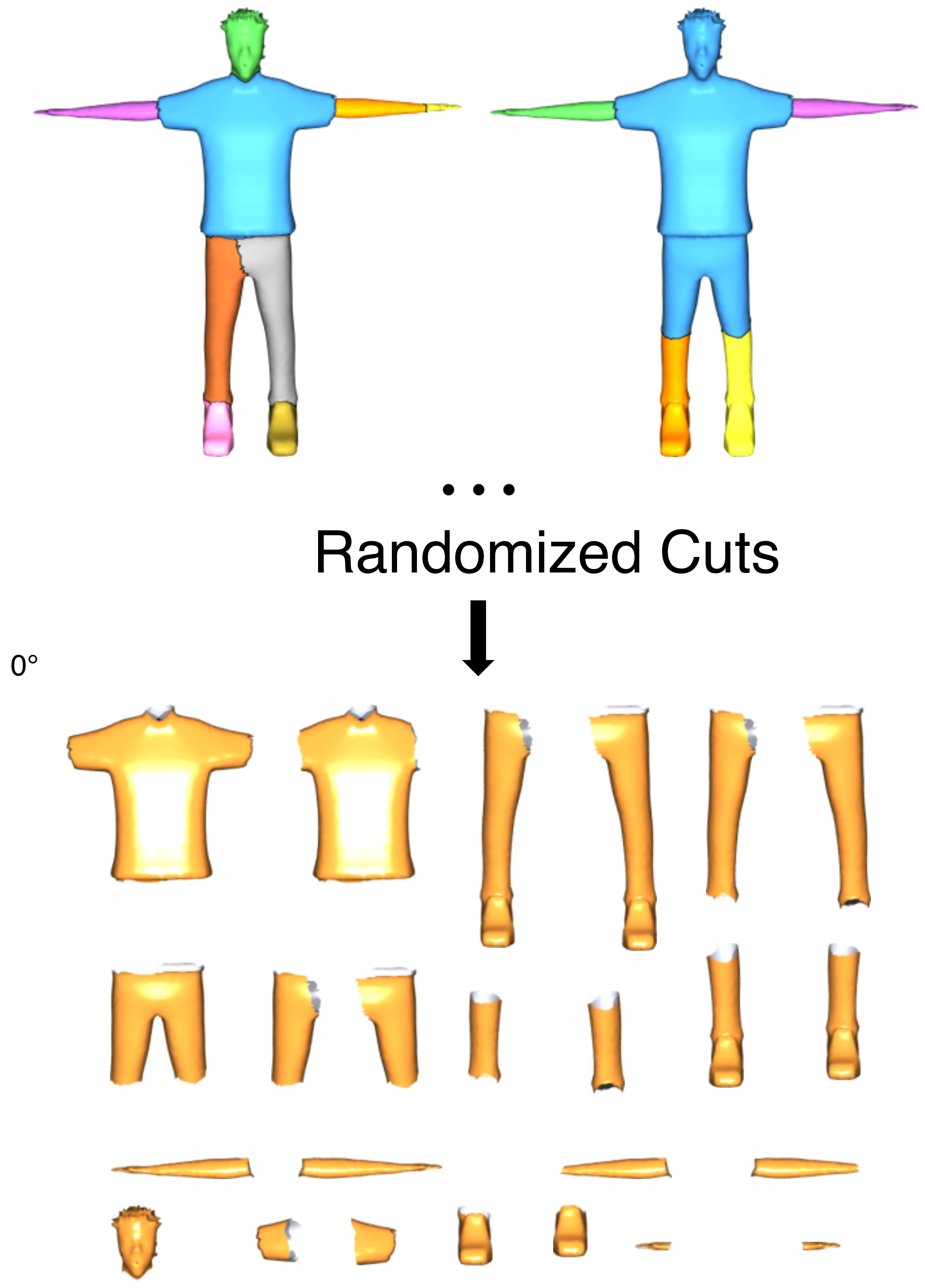
[Huang et al 11]

Segmentations: subsets of initial segments obtained from randomized segmentations

Segmentation constraints: each point is in exactly one segment

$$|\text{cover}(p)| = 1, \quad \forall p \in W$$

The set of initial segments that cover point p



Combinatorial optimization

[Huang et al 11]

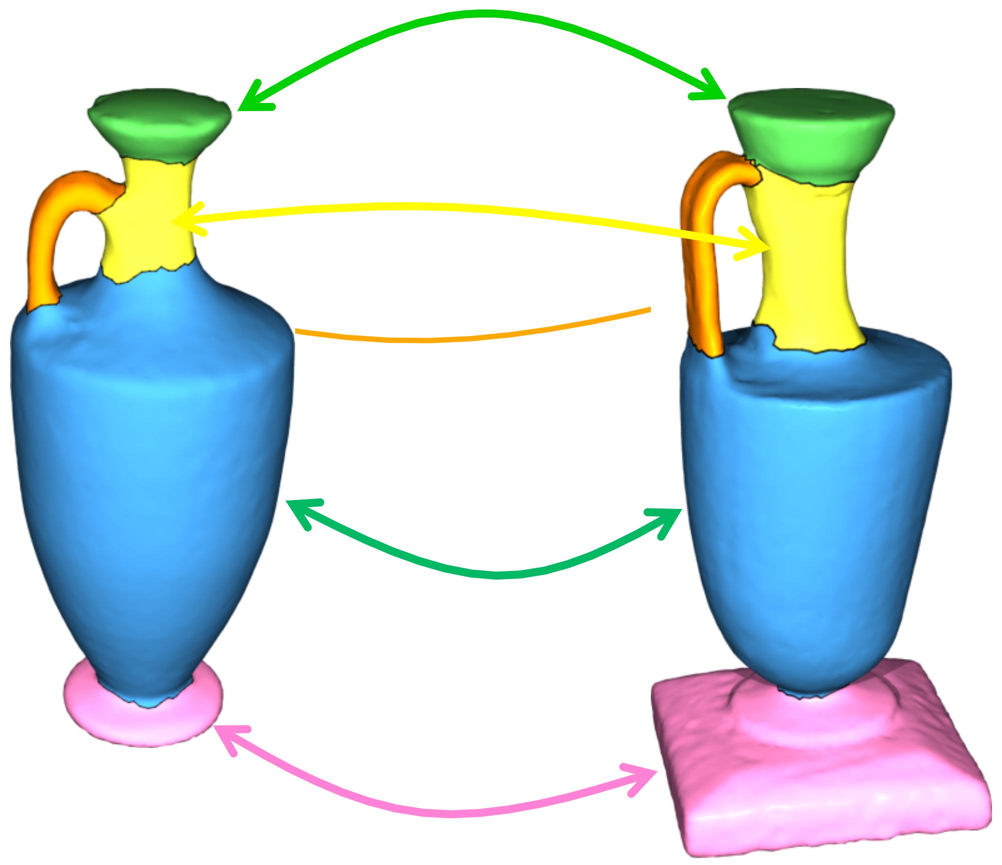
$$\max_{S_1,S_2,\mathcal{M}_{12},\mathcal{M}_{21}} \sum_{i=1}^{2} \sum_{s\in S_i} \overline{w}_s + \sum_{ij\in\{12,21\}} \Big(\lambda \sum_{c\in\mathcal{M}_{ij}} \overline{w}_c + \mu \sum_{(c,c')\in\mathcal{A}_{ij}} \overline{w}_{(c,c')}\Big)$$

s.t.

$$|\text{cover}(p)| = 1, \quad \forall p \in \mathcal{P}_i, \quad 1 \leq i \leq 2,$$

$$\mathcal{M}_{ij} \in \text{Mapping}(\mathcal{S}_i \times \mathcal{S}_j), \quad ij \in \{12, 21\}$$

linear programming relaxation

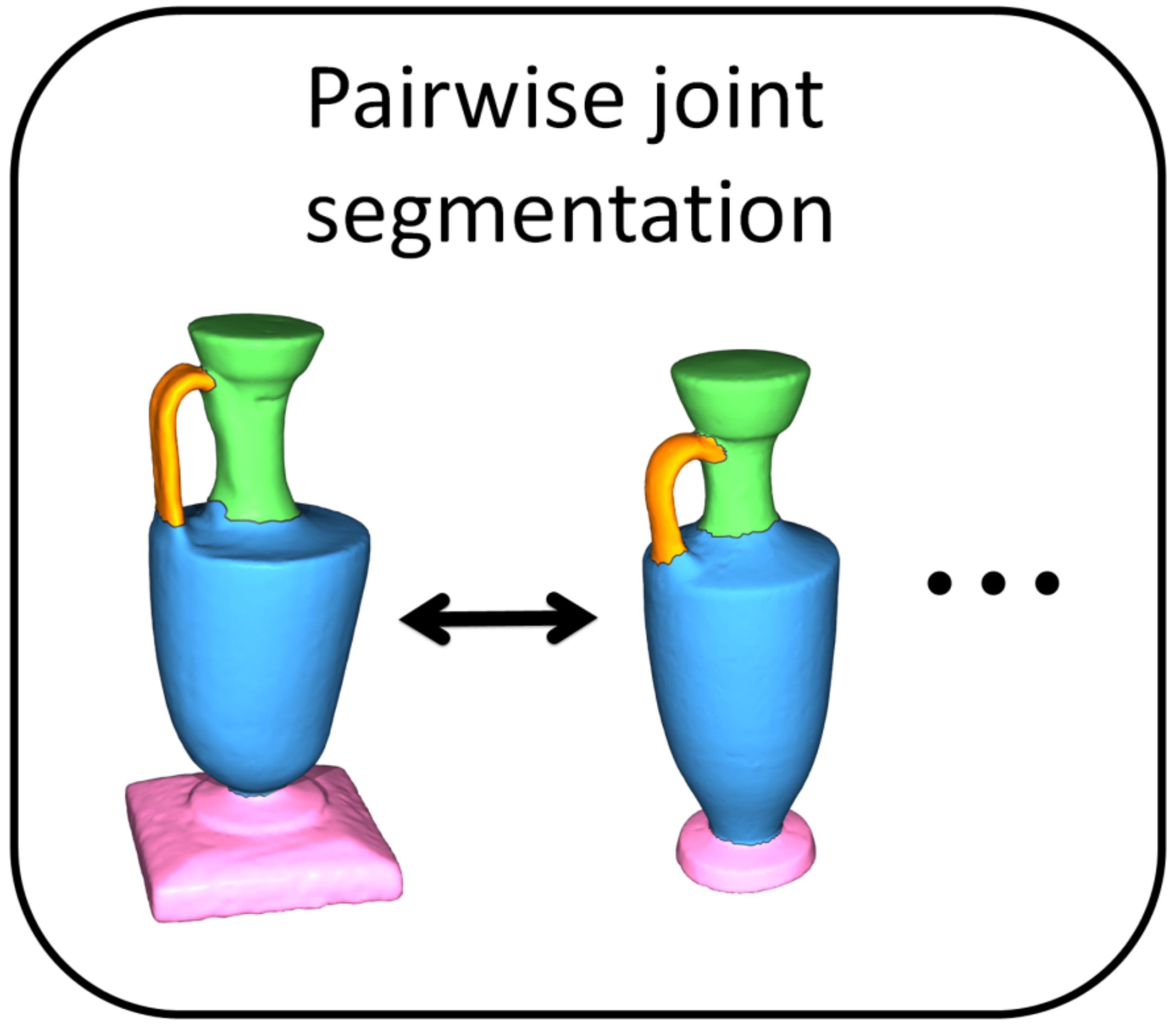


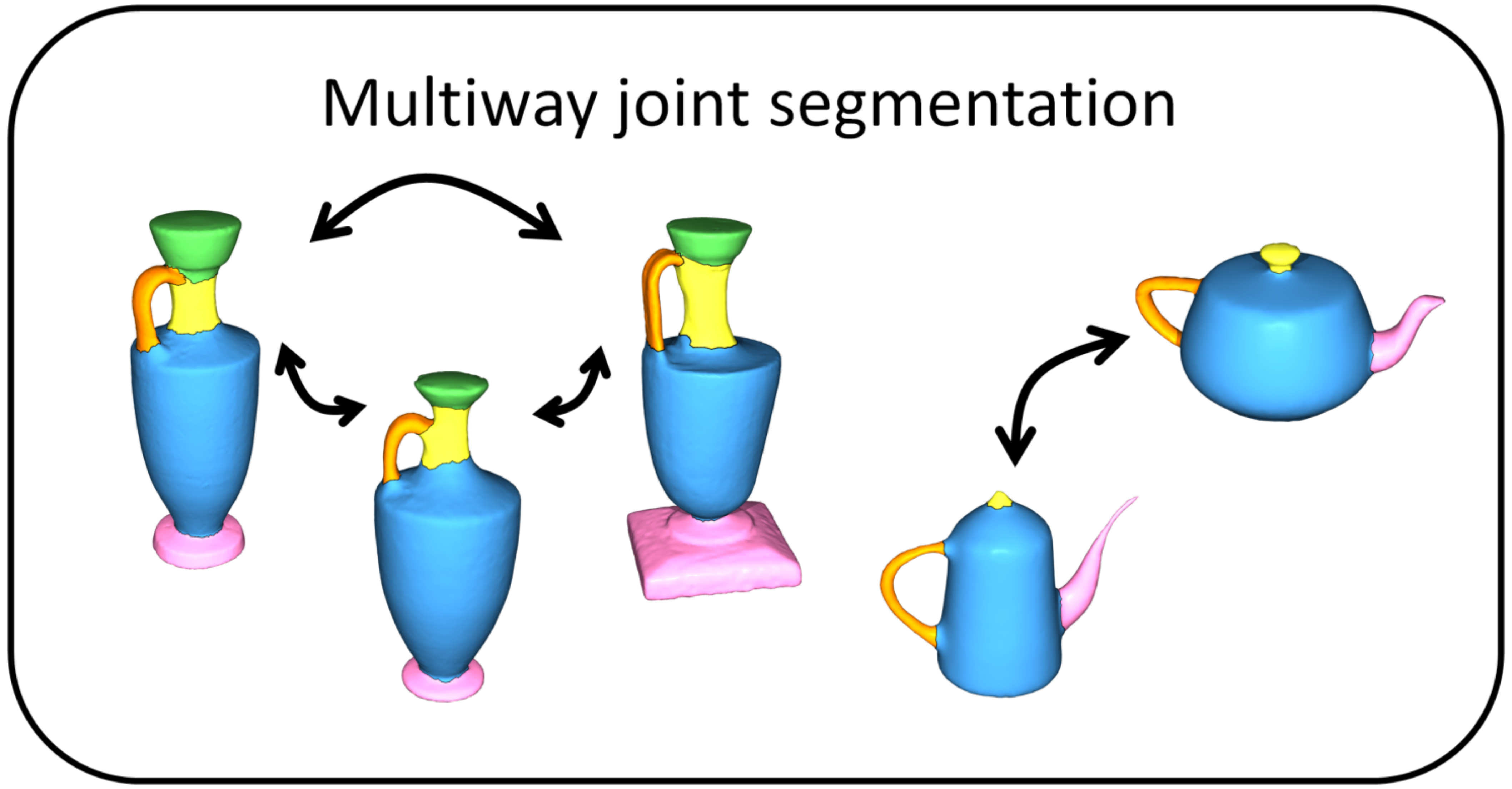
Multi-way joint segmentation

Objective function

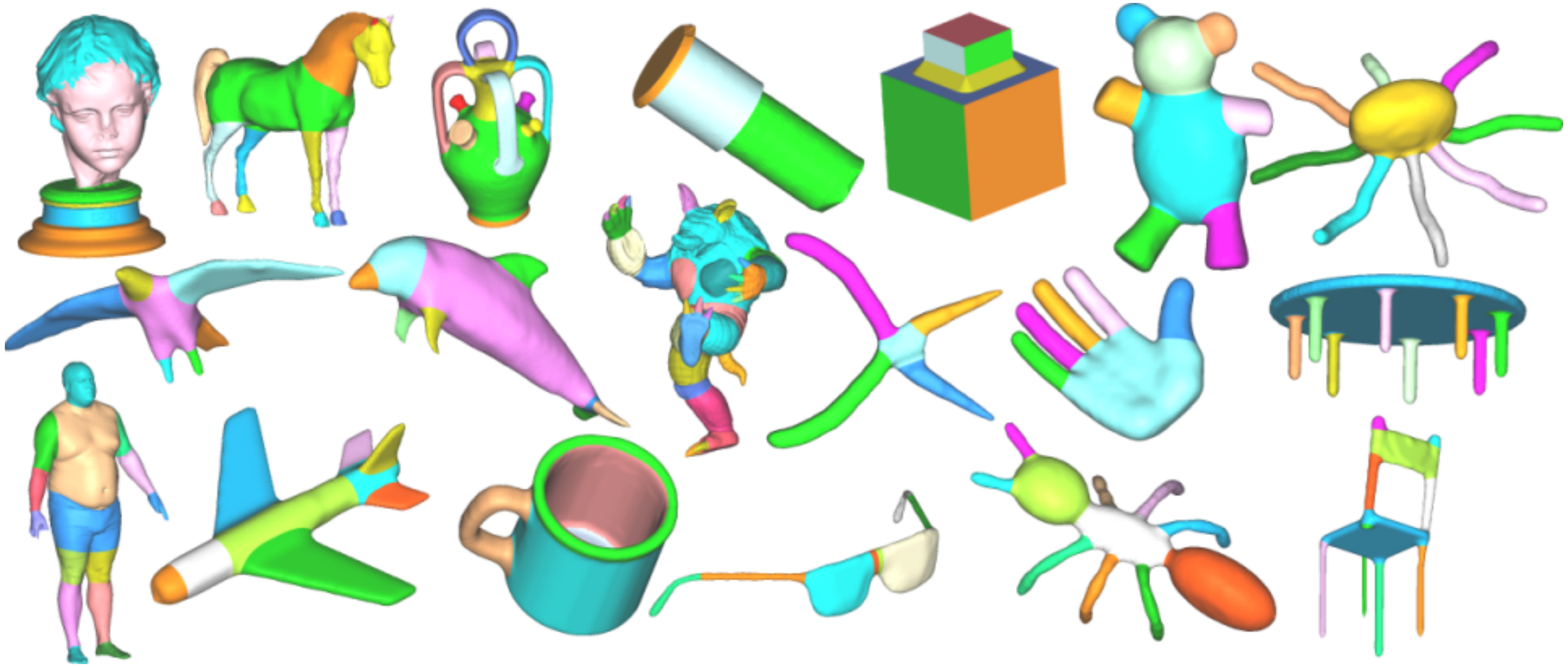
[Huang et al 11]

$$\sum_{i=1}^{n} \mathrm{score}(S_i) + \sum_{(S_i,S_j)\in\mathcal{E}} \mathrm{consistency}(S_i, S_j)$$





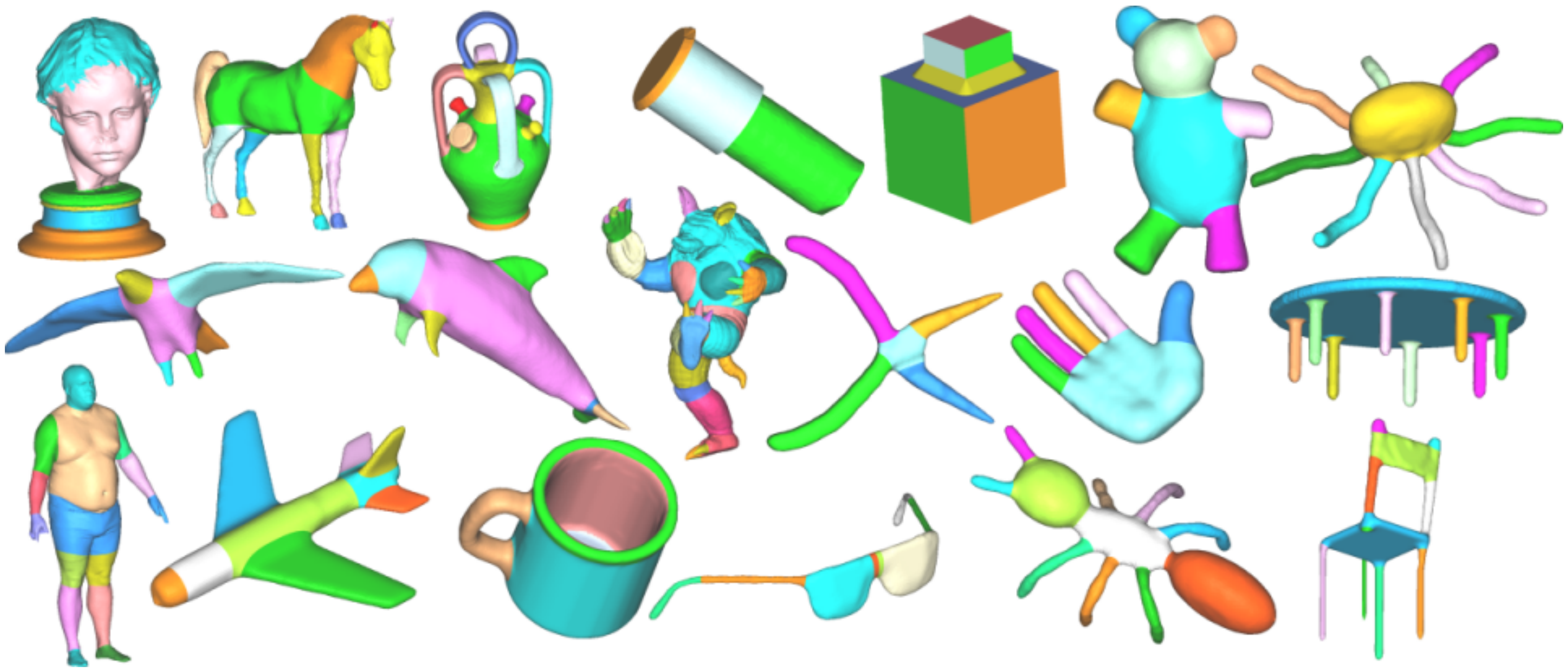
Result on PSB



380 shapes in 19 categories

Manual segmentations for each shape

Result on PSB

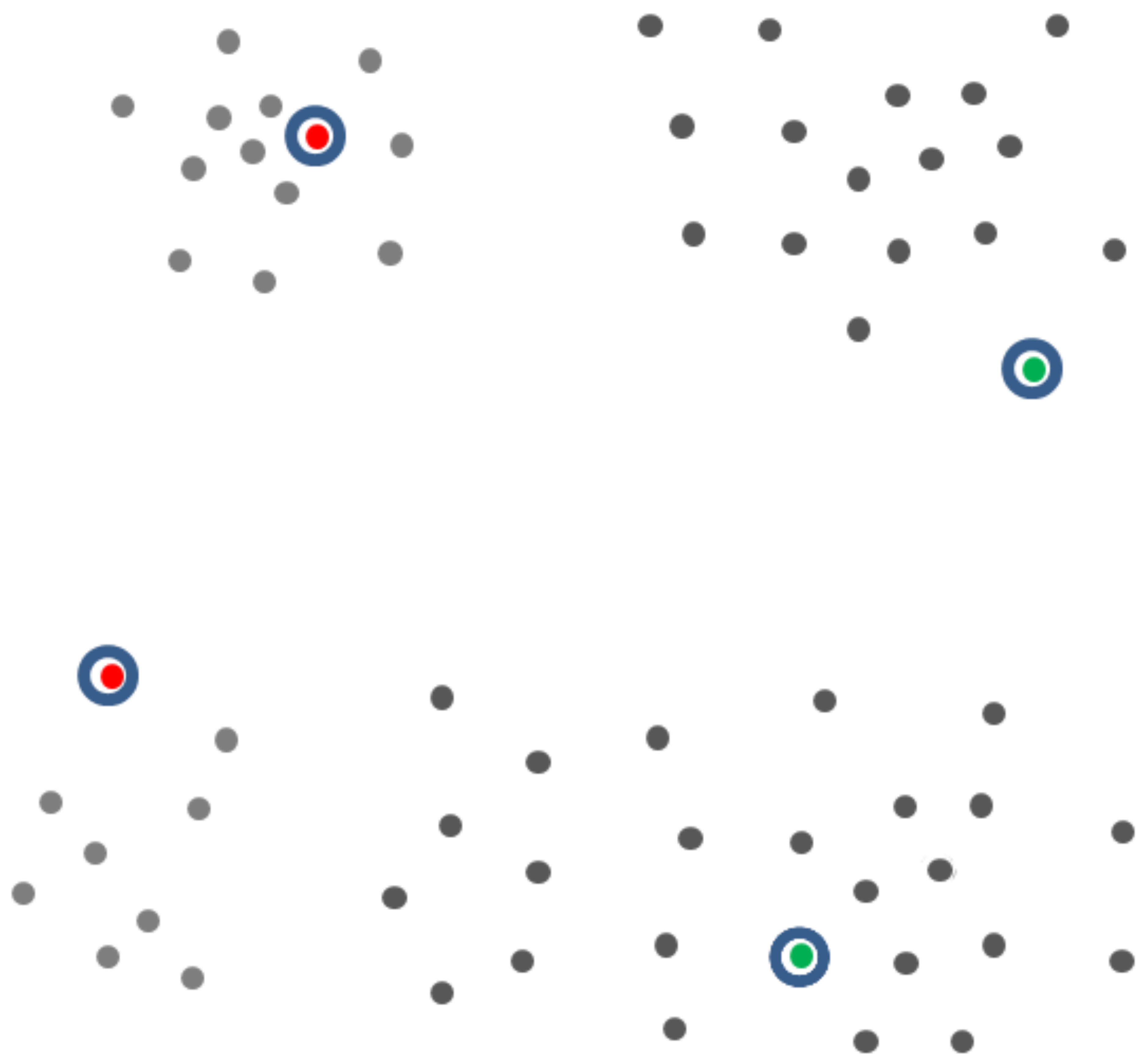


	RCut	SD	SSC	JSS	Supervised
Average	85.6	83.6	88.54	90.8	90.5-92.1

Semi-supervised methods

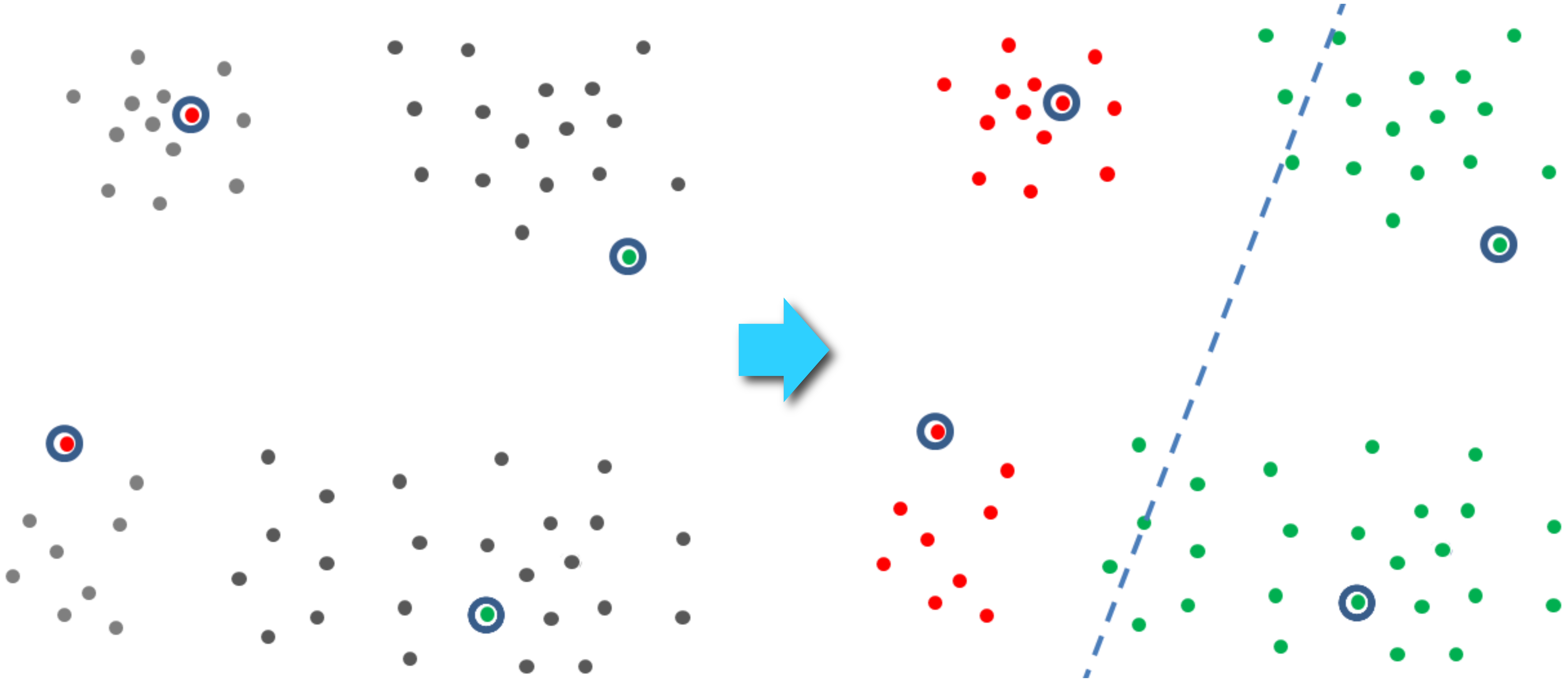
Machine learning

[Wang et al 12]



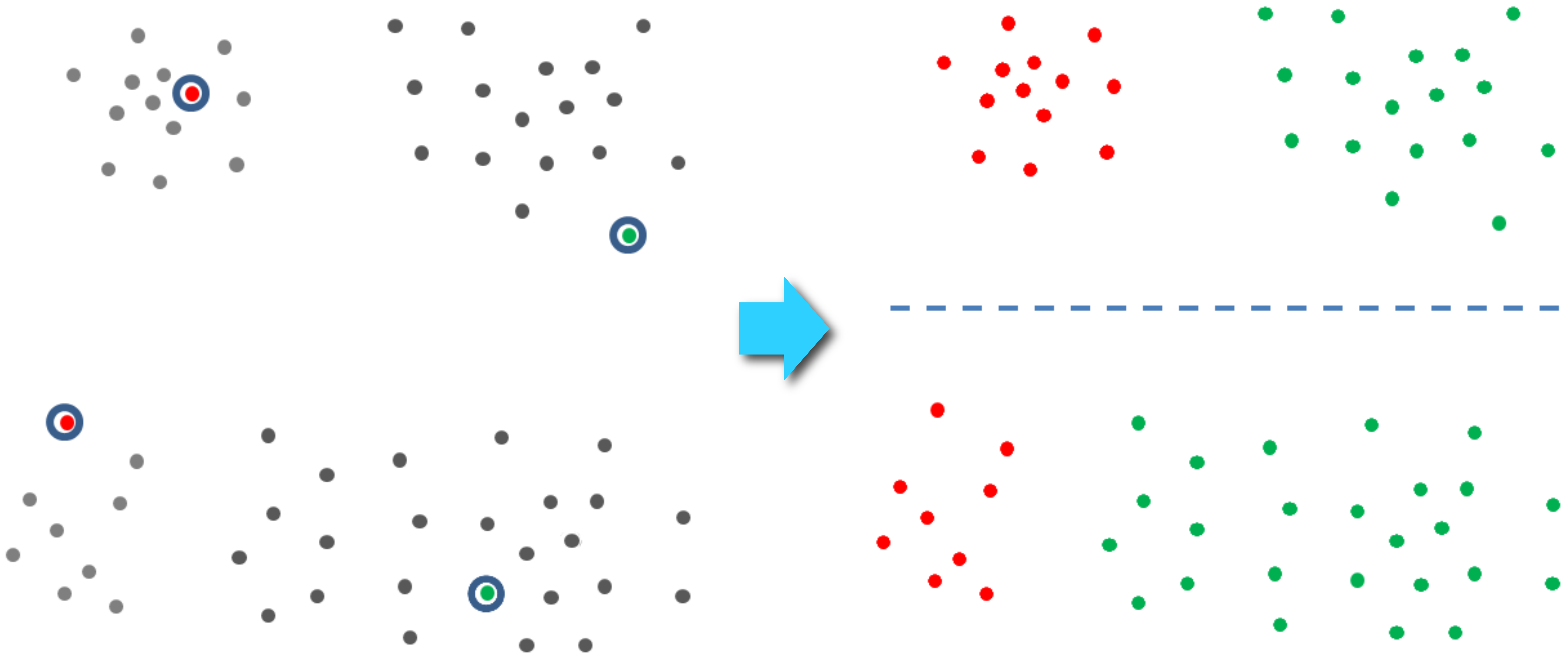
Supervised learning

[Wang et al 12]



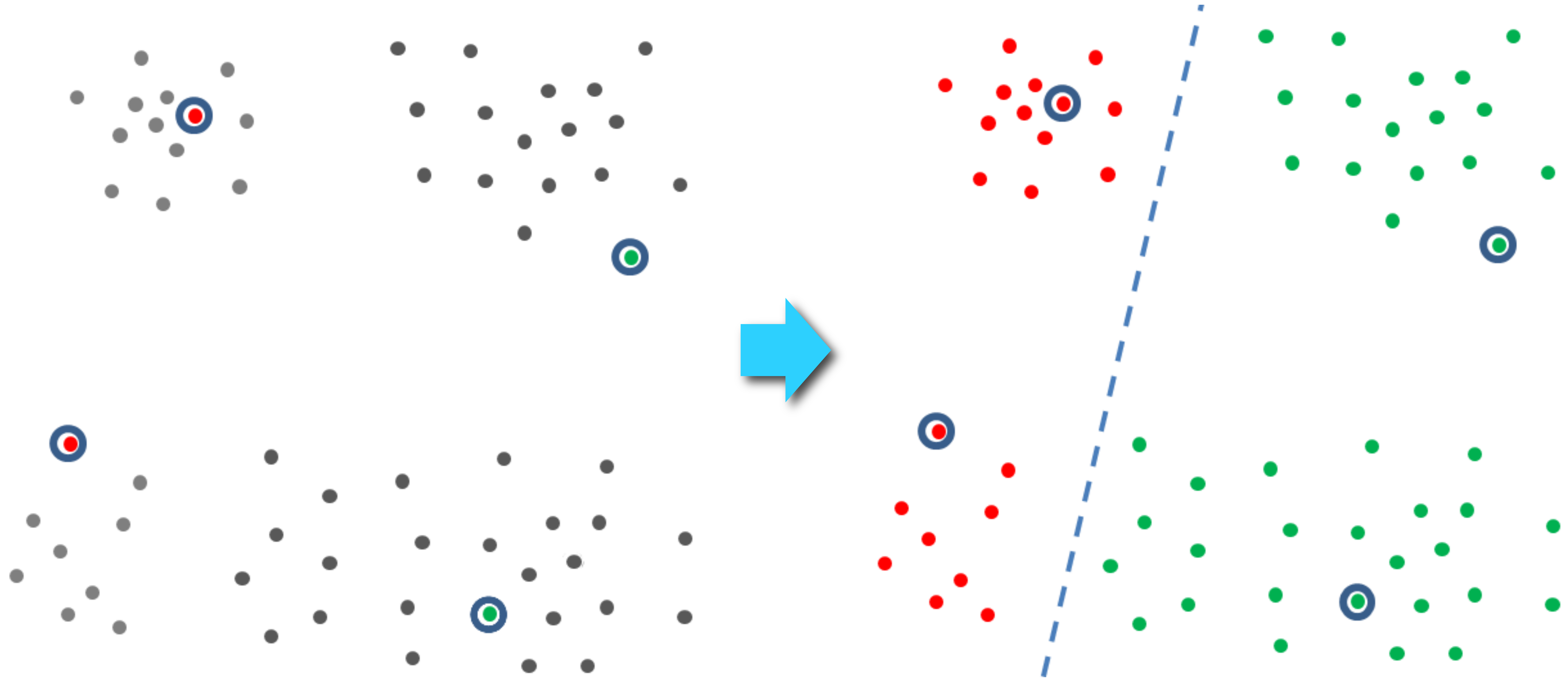
Un-supervised learning

[Wang et al 12]



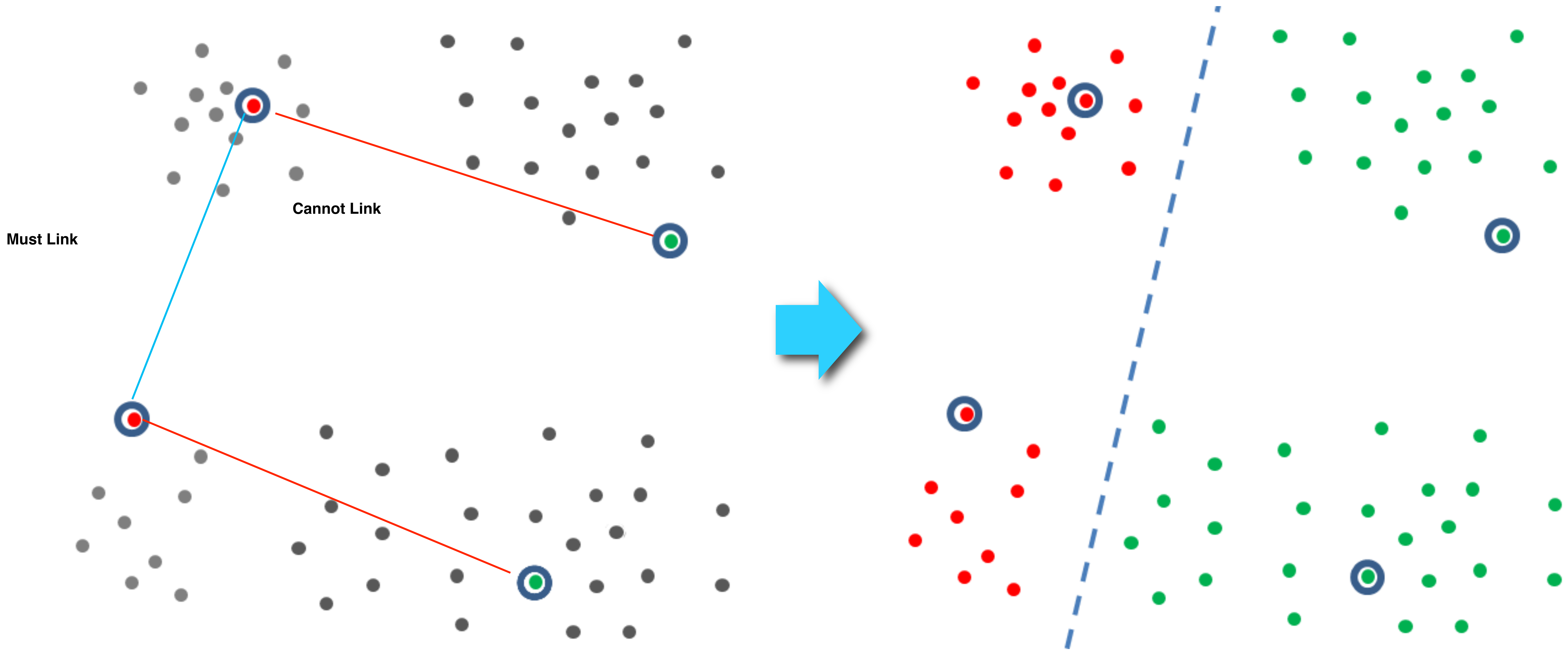
Semi-supervised learning

[Wang et al 12]



Constrained clustering

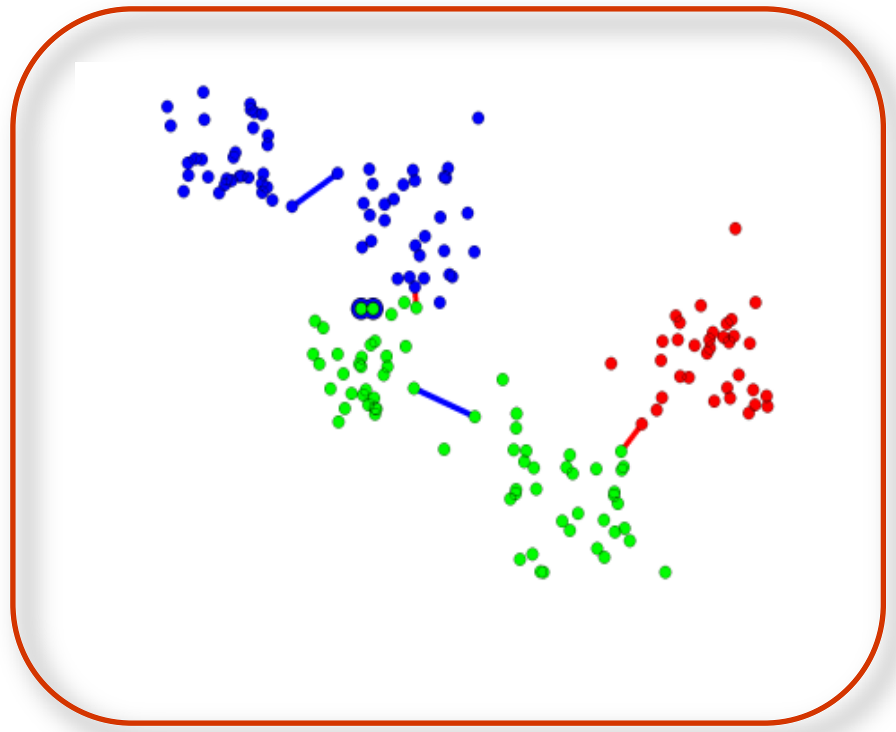
[Wang et al 12]



Algorithm

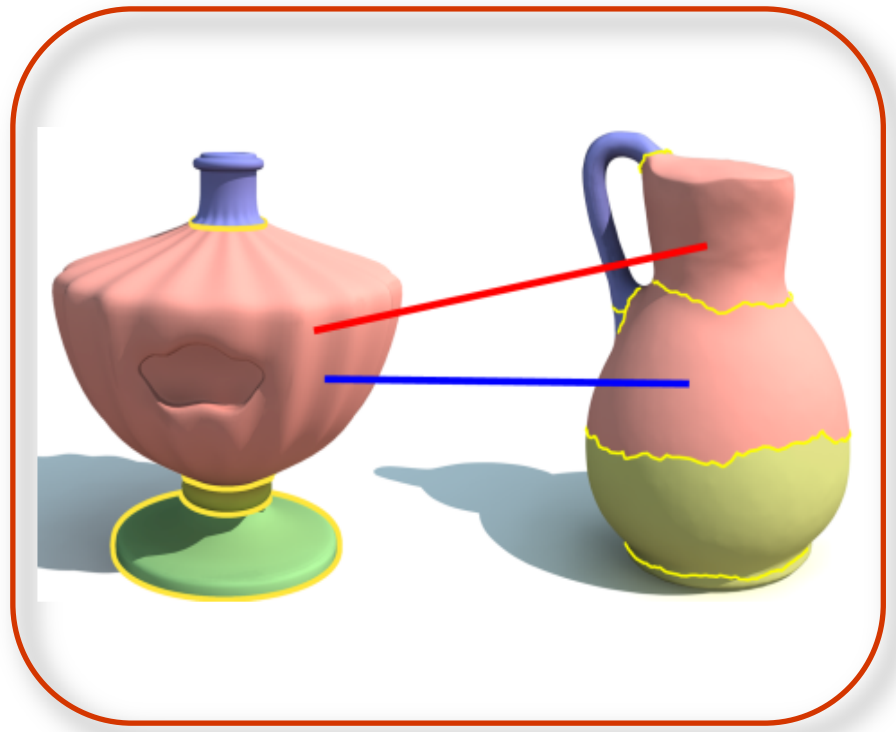
[Wang et al 12]

Initial Co-segmentation



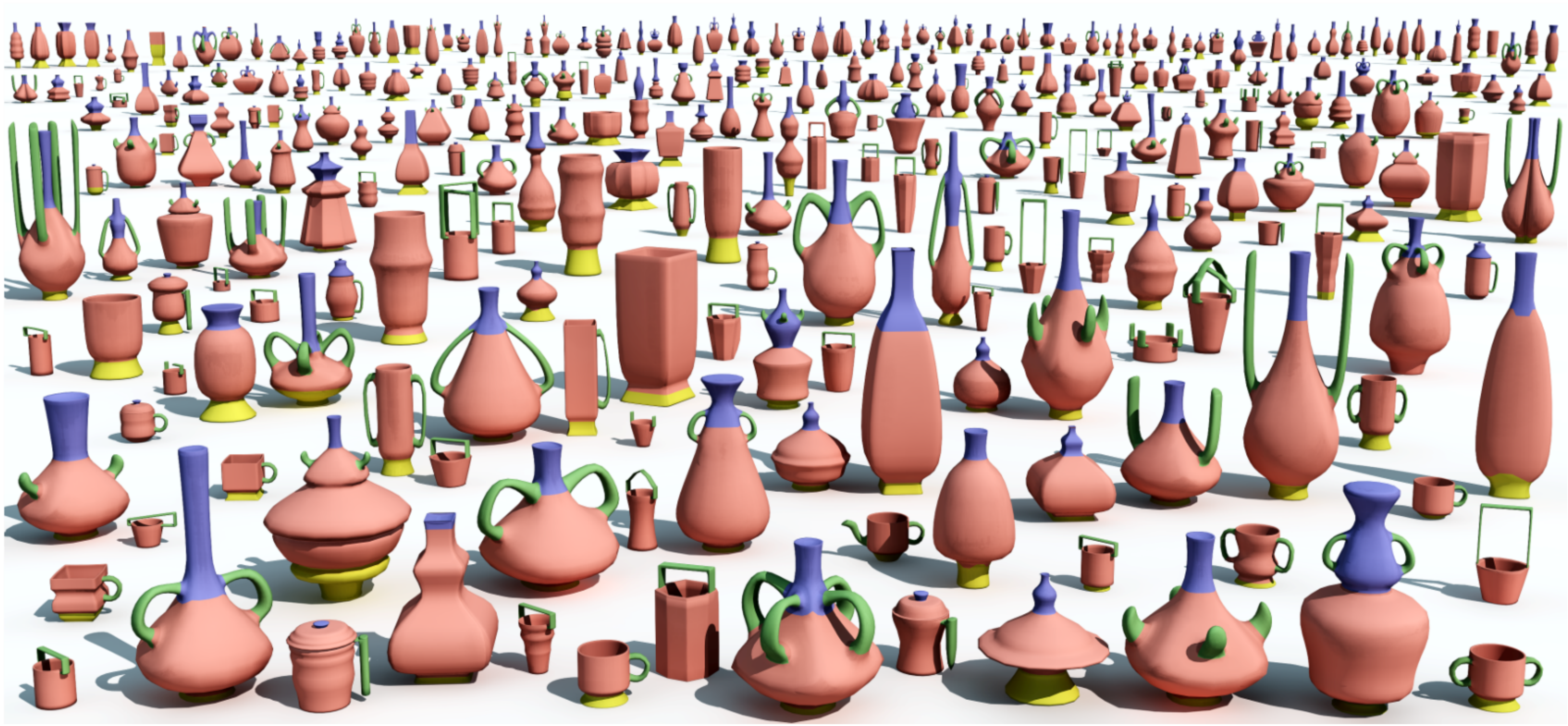
Constrained Clustering

Final result



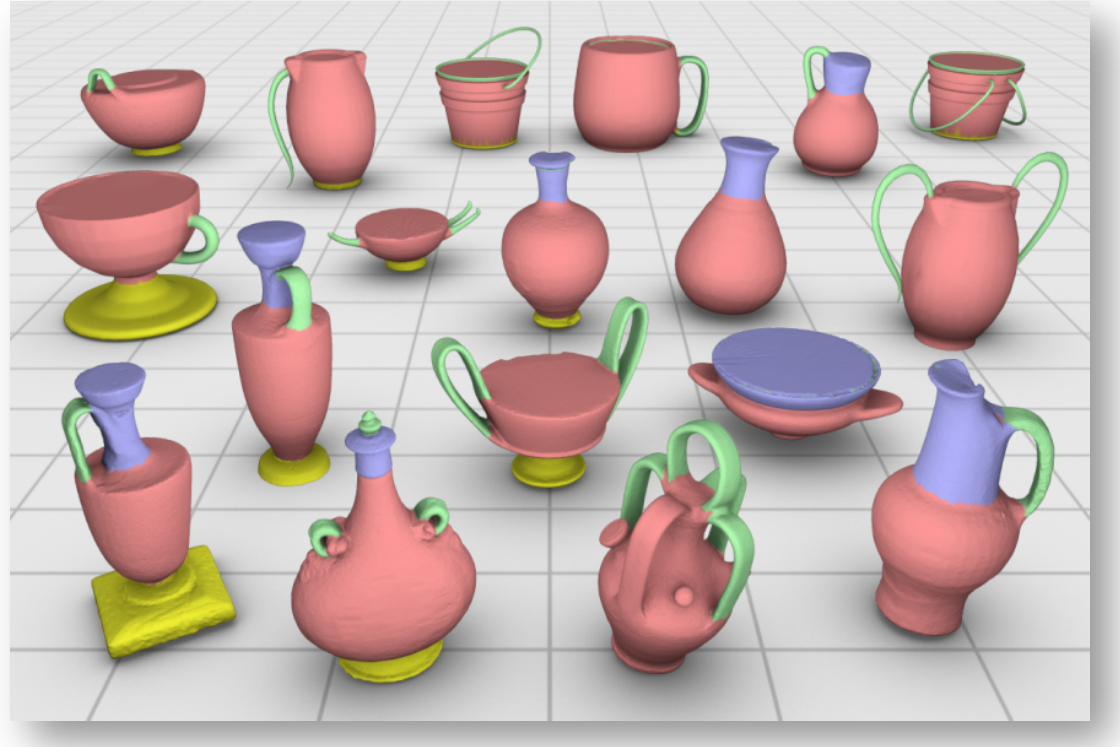
Active Learning

Effective on large data sets

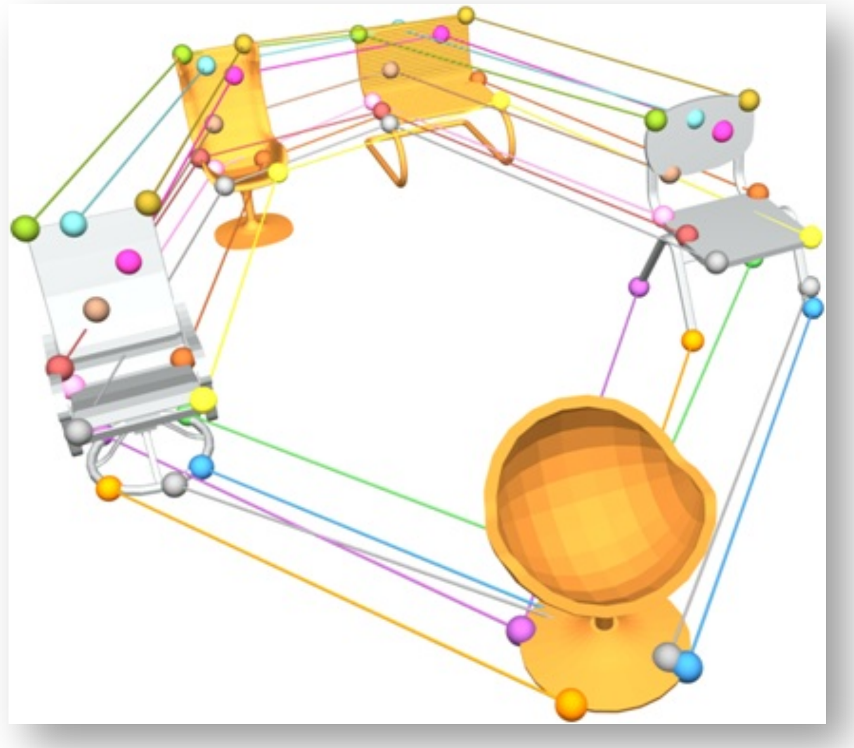


300 shapes

Outline



Co-segmentation

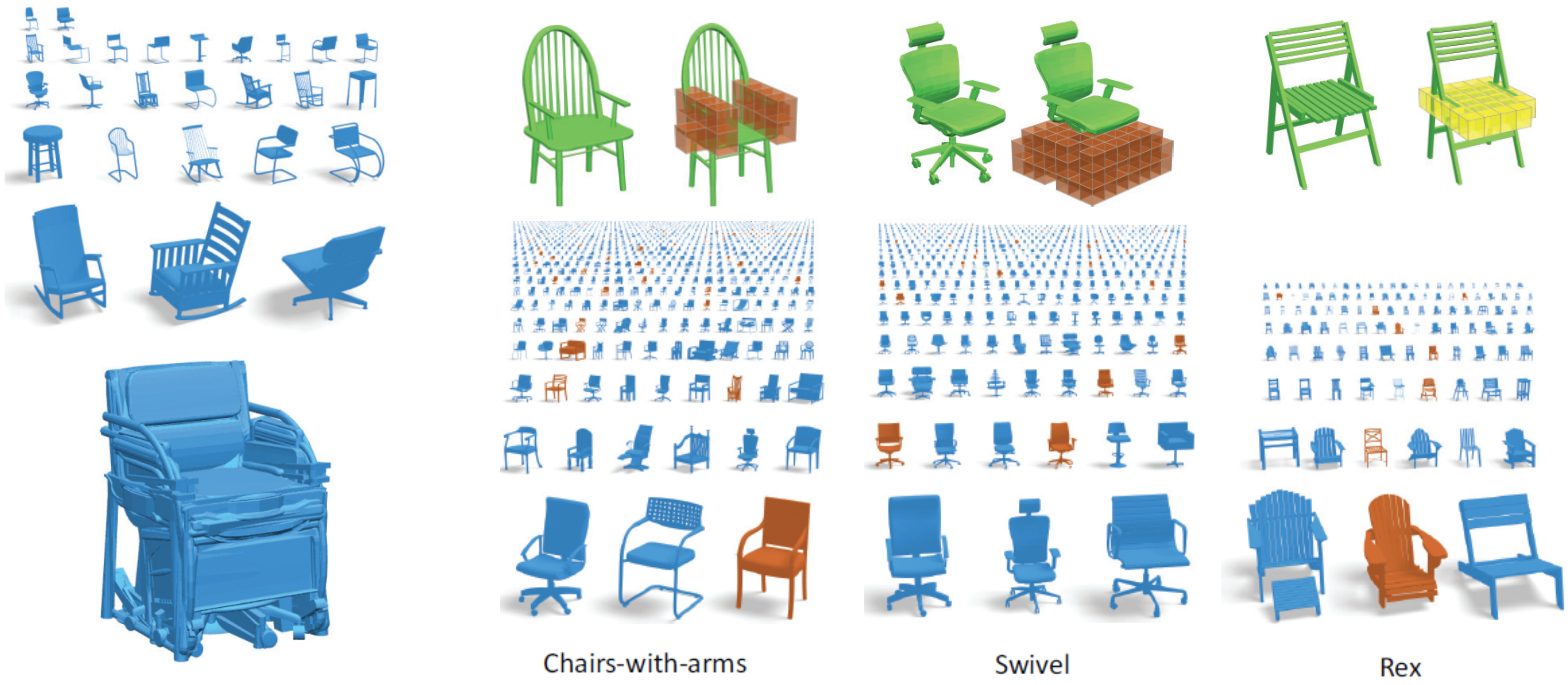


Joint matching

Other types of co-analysis

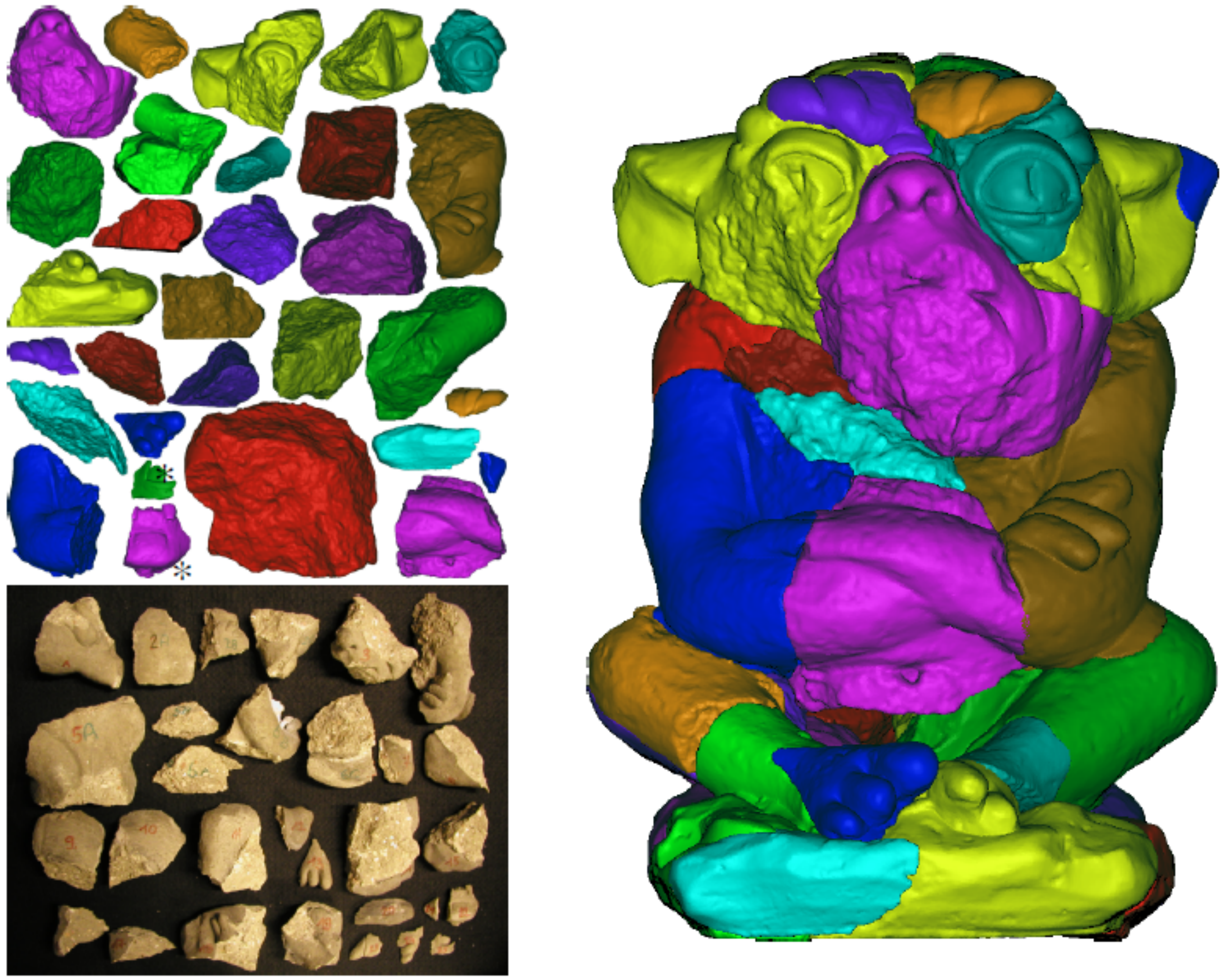
Fine-grained classification

[Huang et al 13]



Assembling fractured pieces

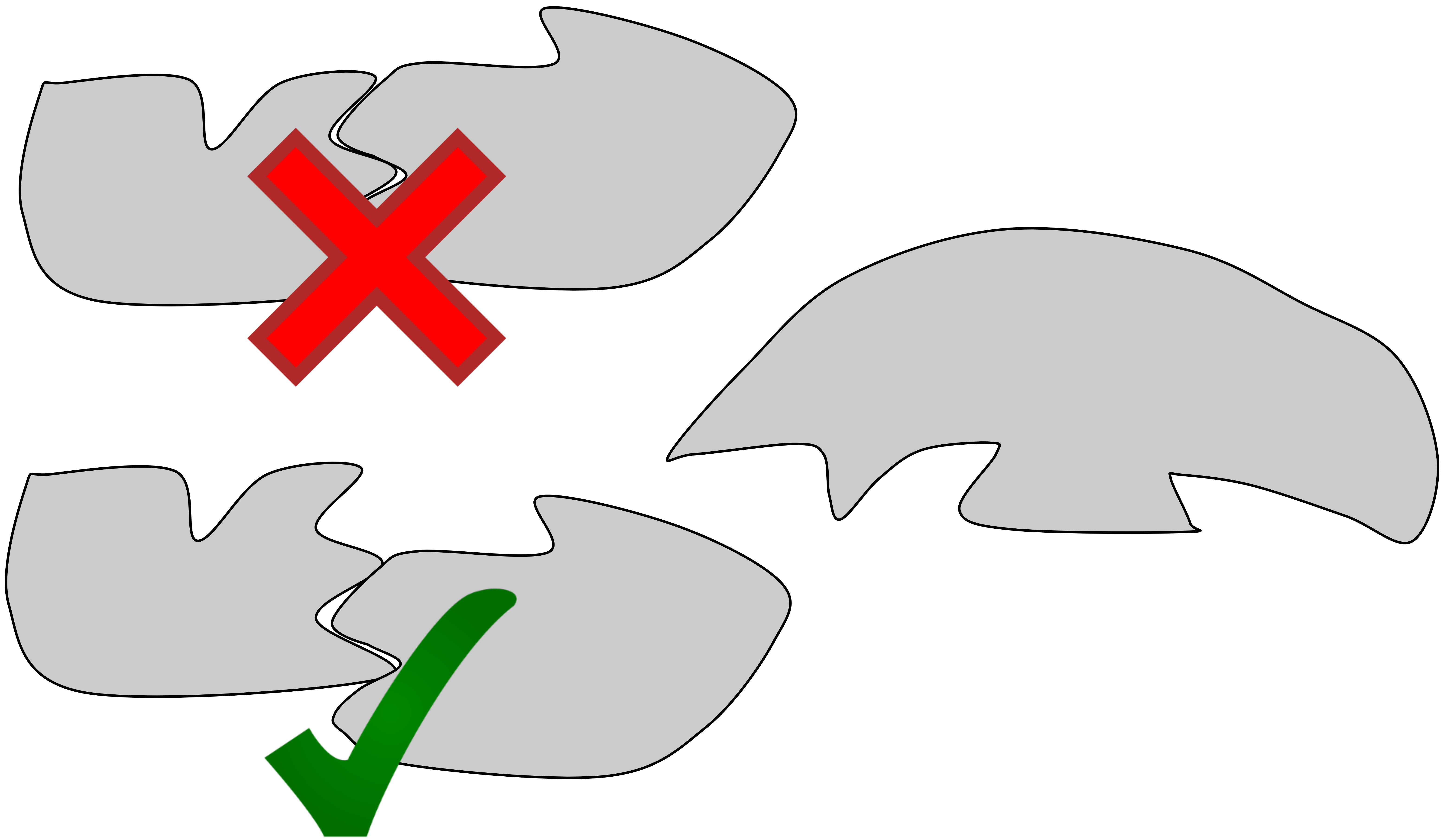
Manual assembly
(Ephesus, Turkey)



Computer assembly
[Huang et al 06]

Structure from Motion

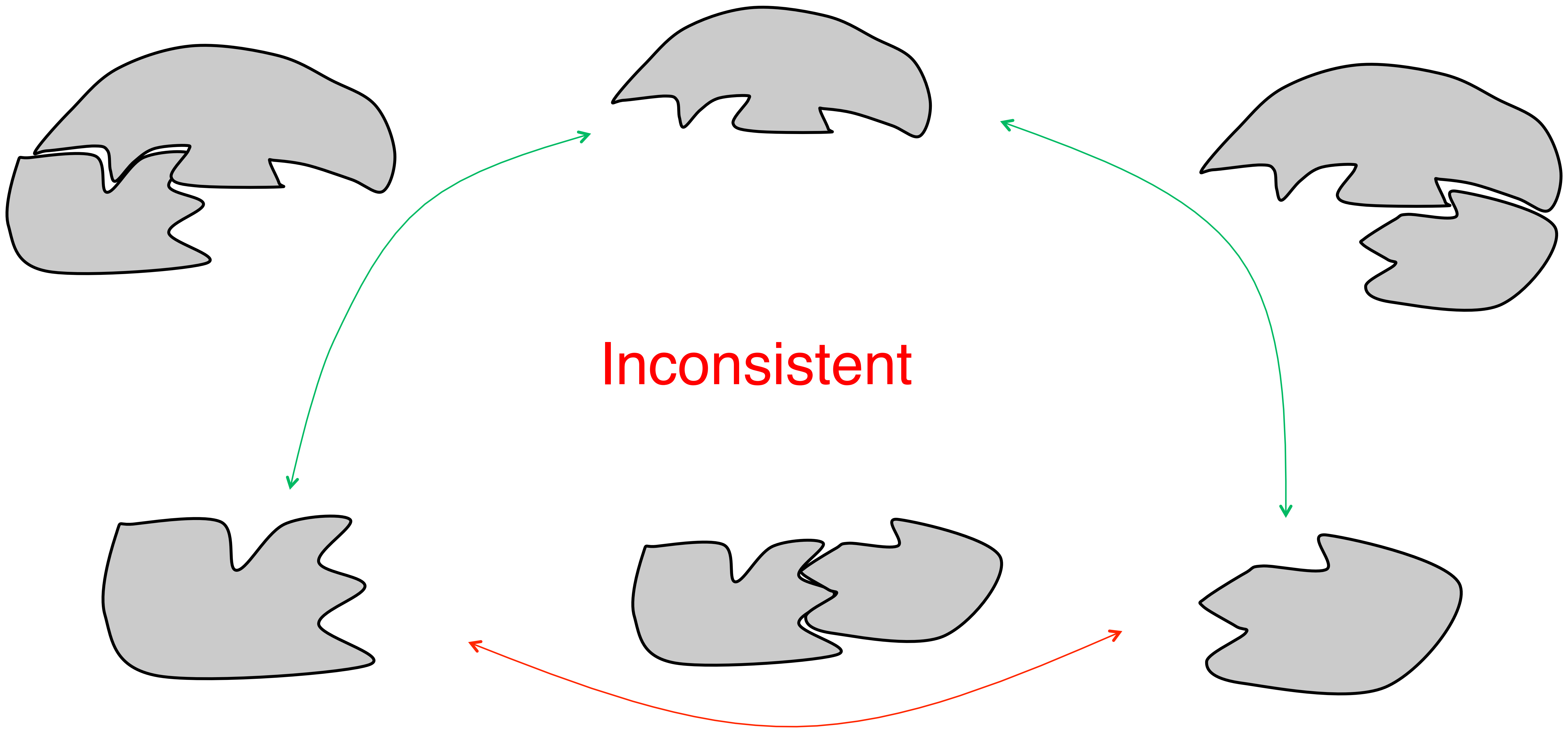
Shape matching



Cycle-consistency

Global compatibility criterion:

Composition maps along cycles are identity maps



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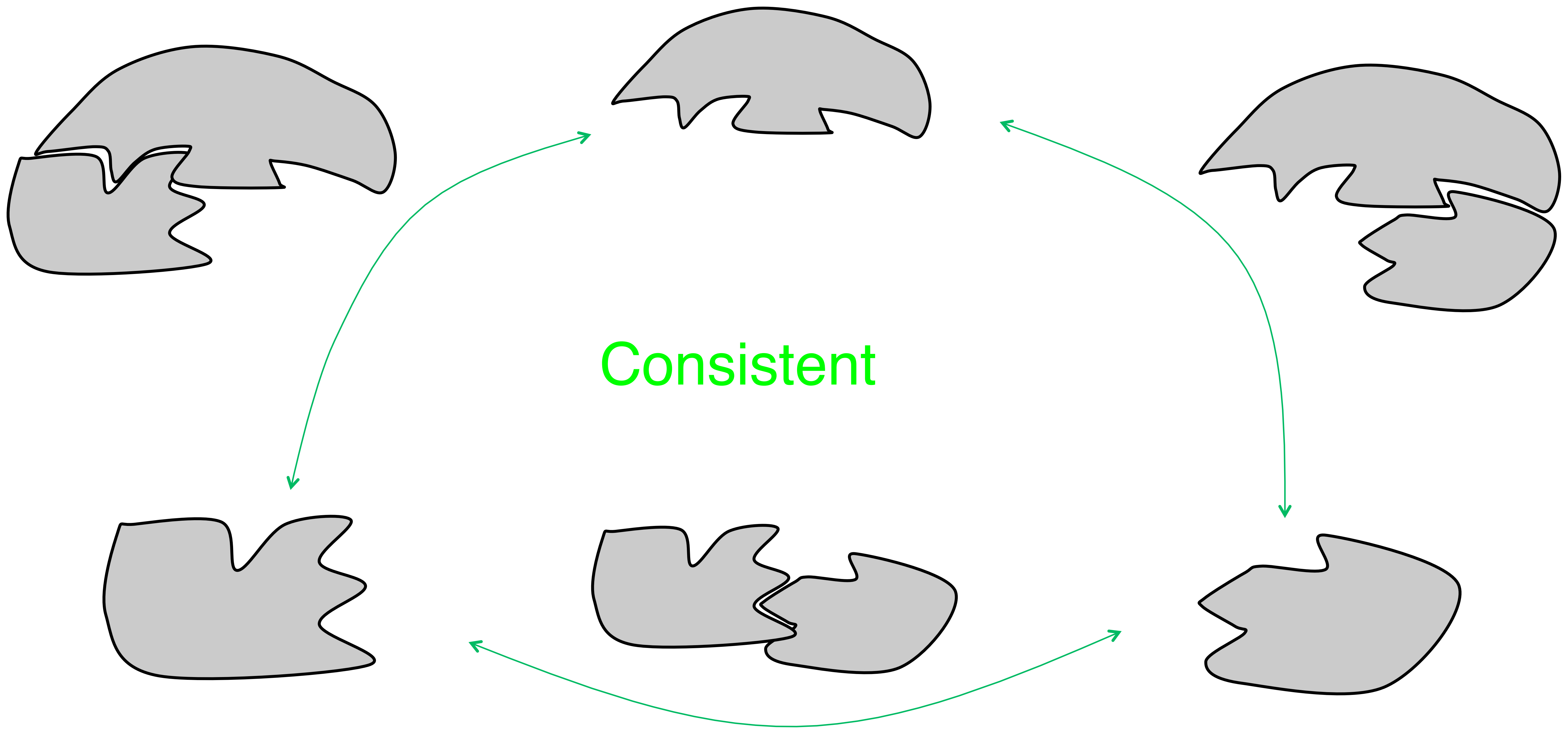
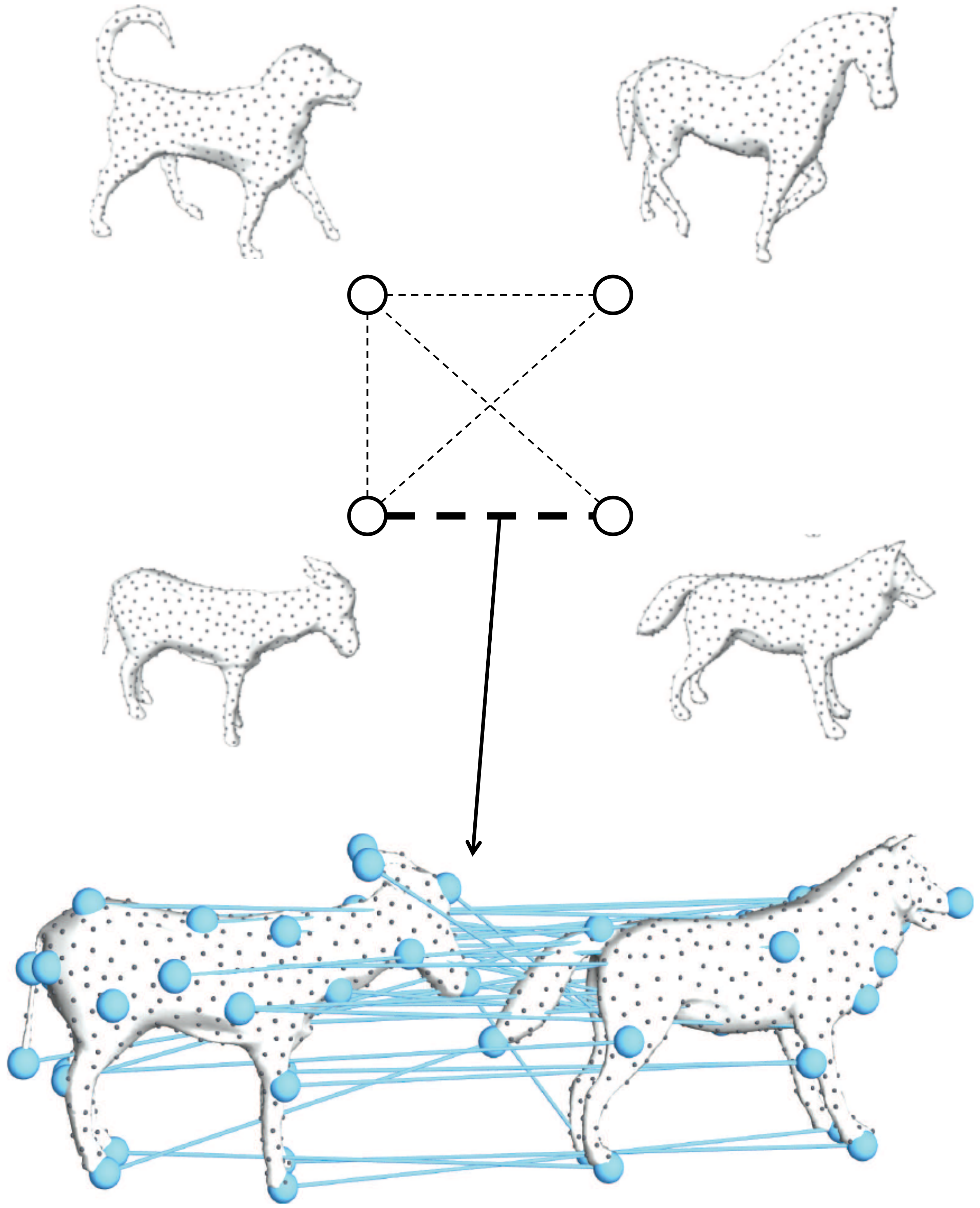


Popular formulation

Input:

A set of shapes

A few initial maps computed in isolation along a shape graph *G* using an existing method



Popular formulation

Input:

A set of shapes

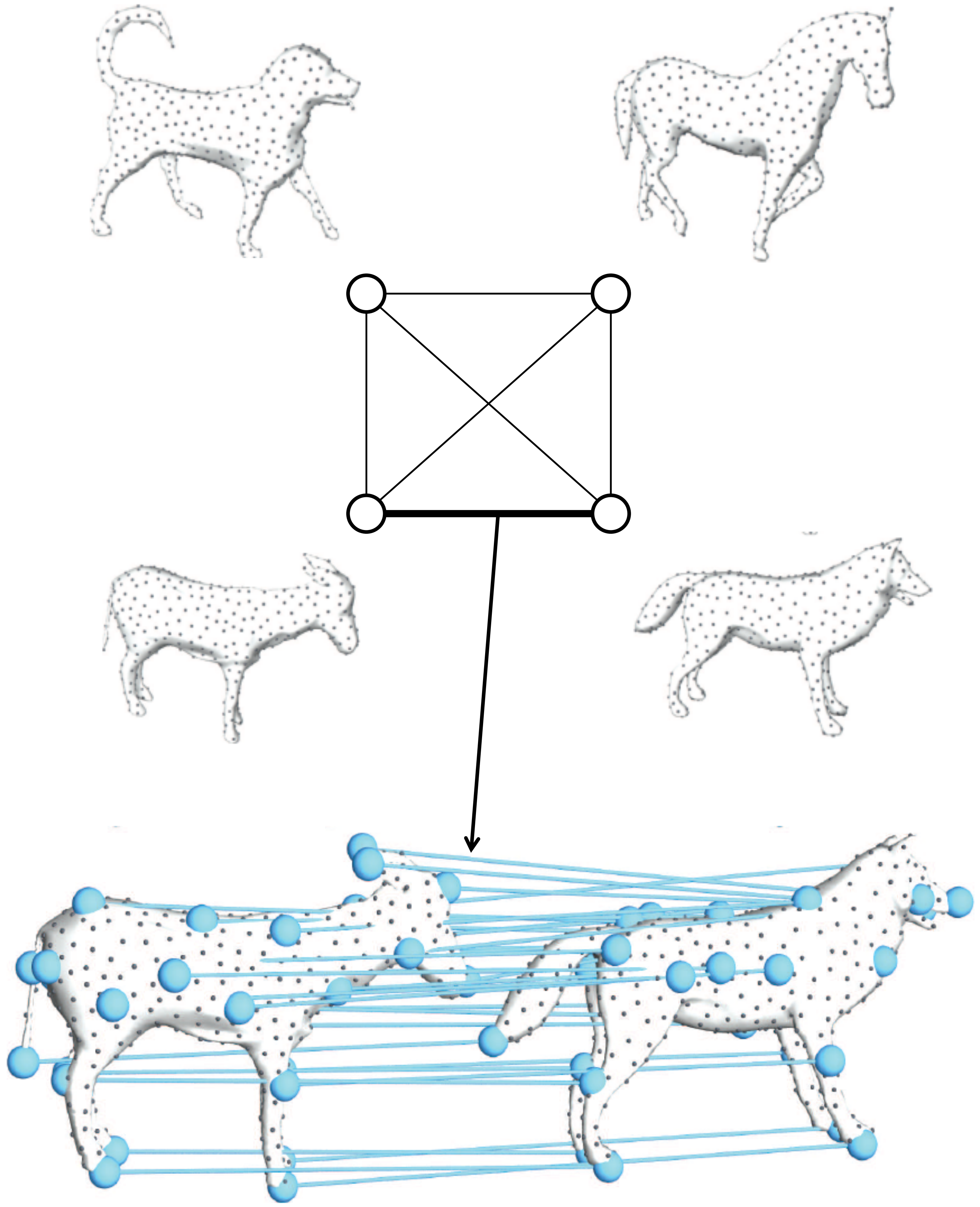
A few initial maps computed in isolation along a shape graph *G* using an existing method

Output:

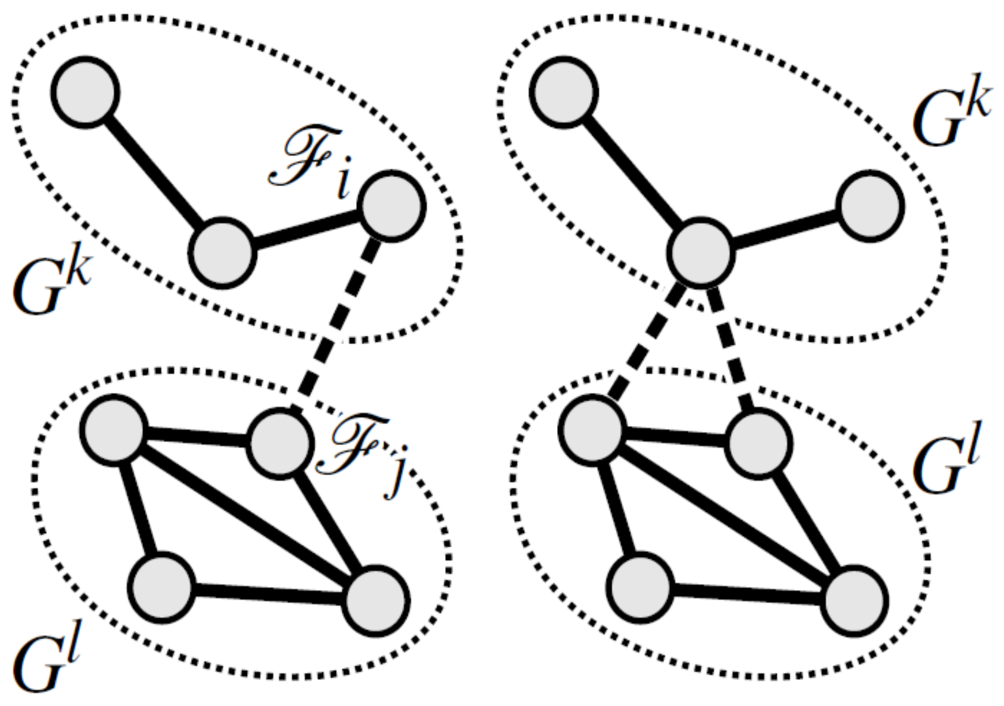
Maps between all pairs of objects

Cycle-consistent

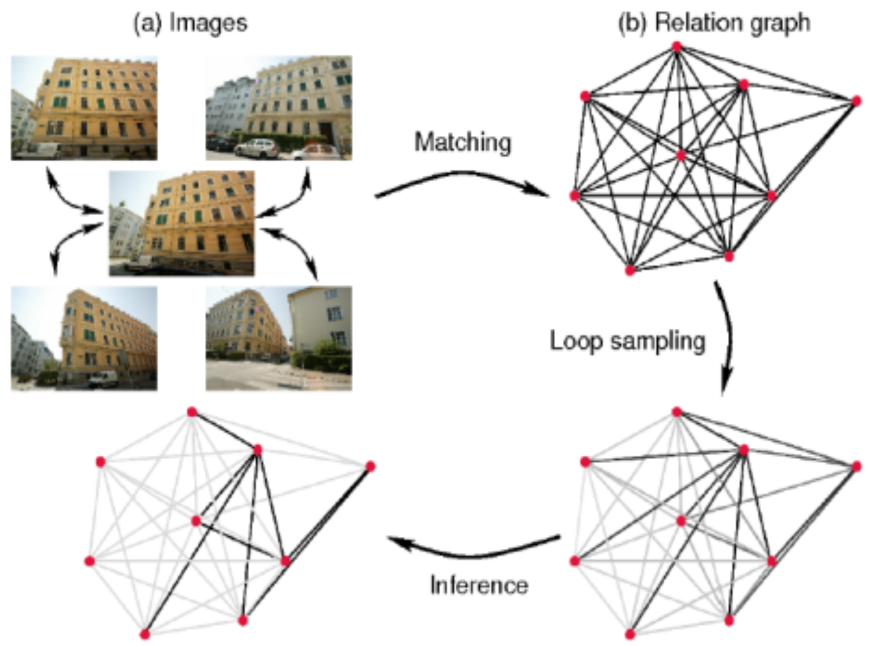
Close to the input maps



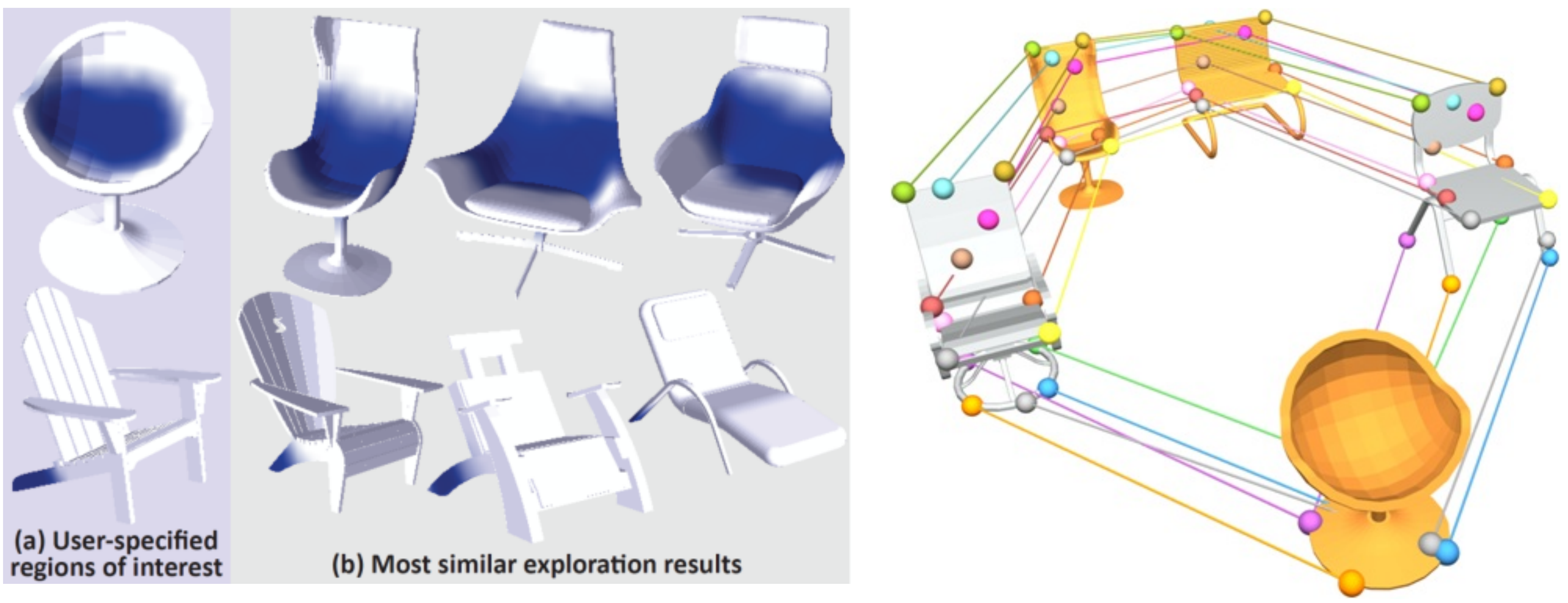
Different optimization strategies



Spanning tree optimization
[Huber 02, Huang et al 06]



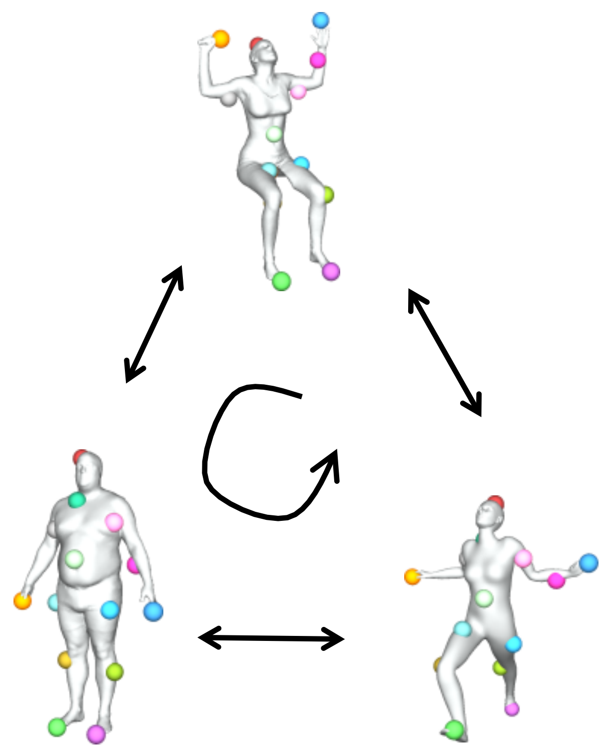
Detecting Inconsistent Cycles
[Zach et al. 2010, Nguyen et al. 2011]



Spectral techniques [Kim et al 12, Huang et al 12]

Convex optimization

[Huang et al 13]



Cycle-consistent

X º 0

(Positive) semidefiniteness

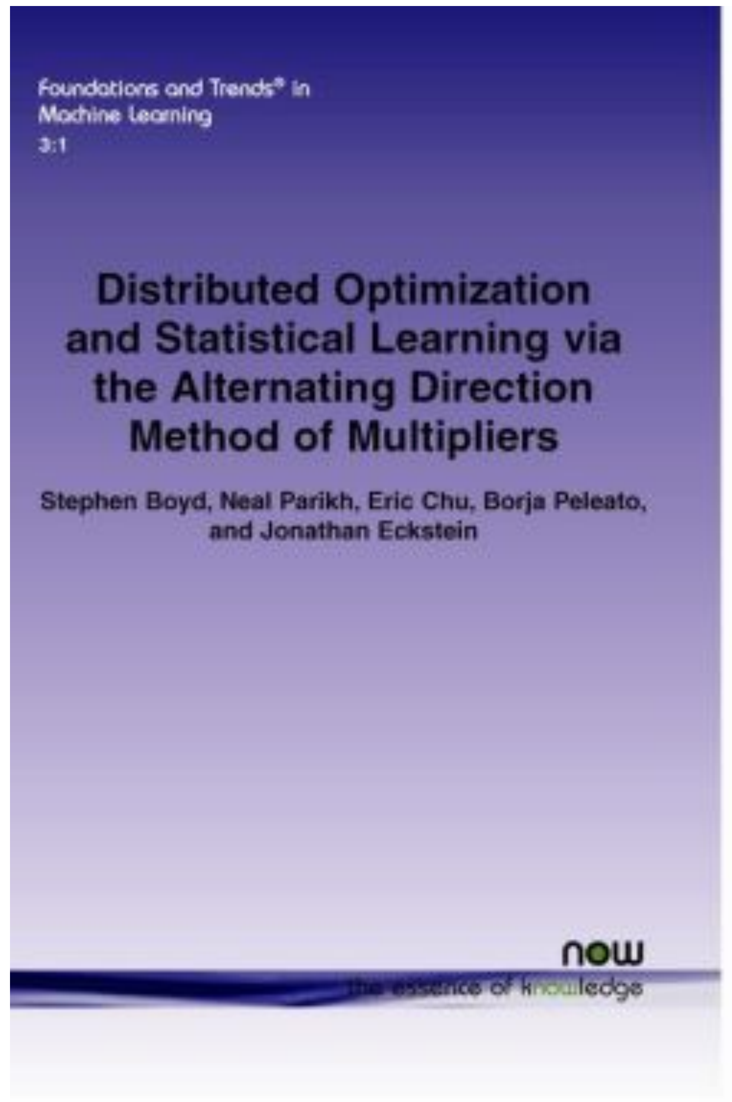
$$X = \begin{pmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{12}^T & I_m & \cdots & \cdots \\ \vdots & \vdots & I_m & X_{(n-1),n} \\ X_{1n}^T & \vdots & X_{(n-1),n}^T & I_m \end{pmatrix}$$

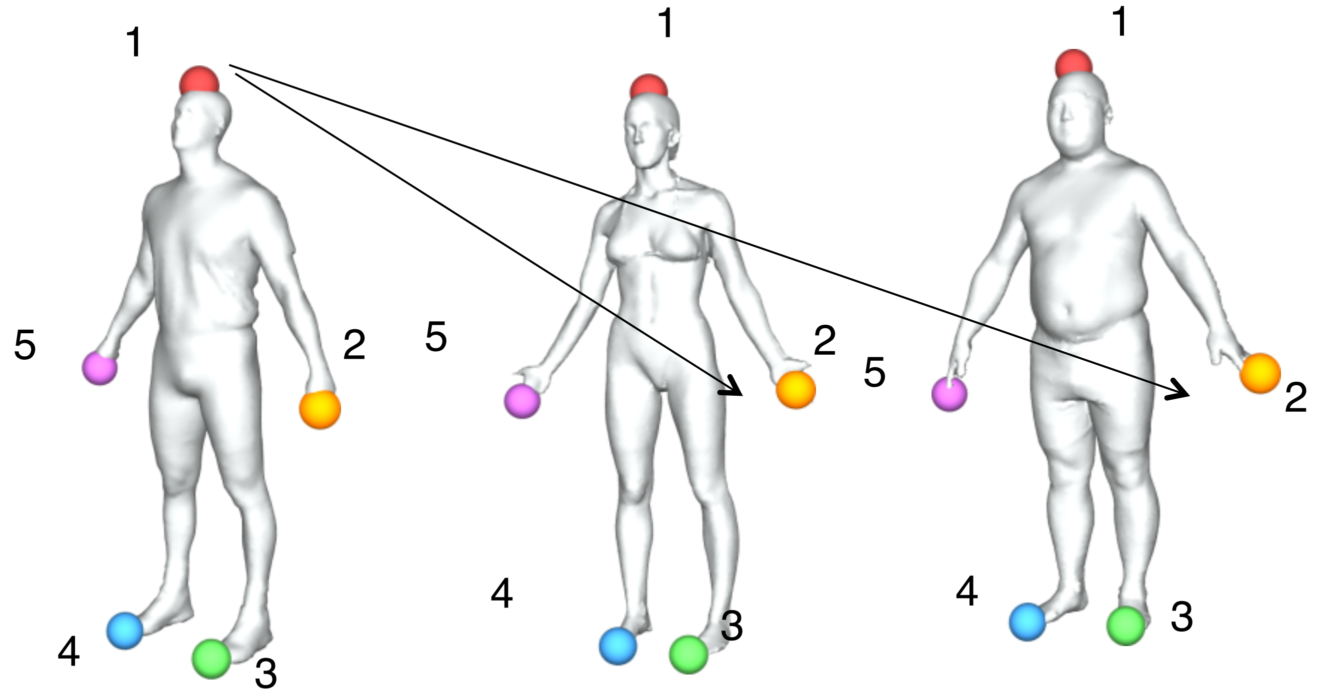
Semidefinite programming

$$\text{minimize} \quad d(X^{\text{input}}; X)$$

$$\text{subject to} \quad X \succeq 0$$

$$\mathcal{A}(X) \leq b$$





Exact recovery condition:

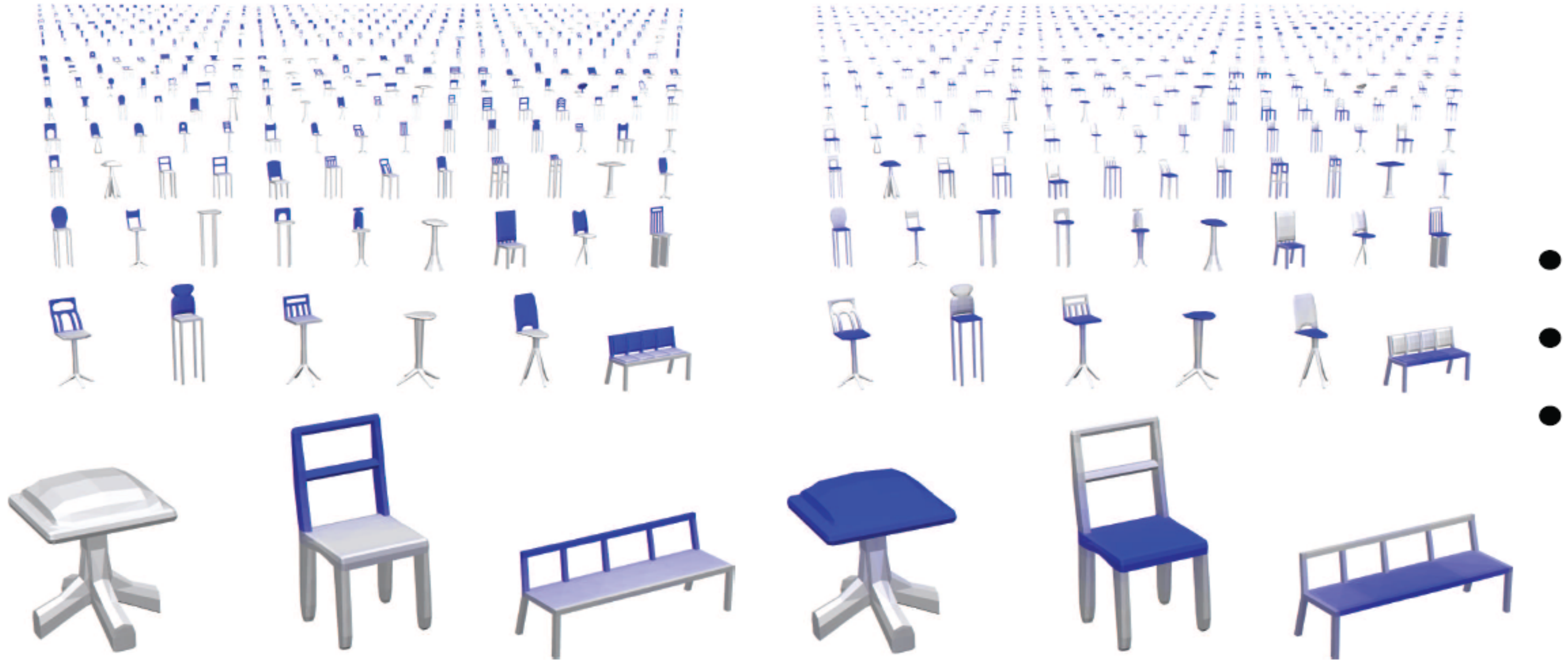
incorrect correspondences% < constant

Man-made shapes (partial similarity)

[Huang et al 14]

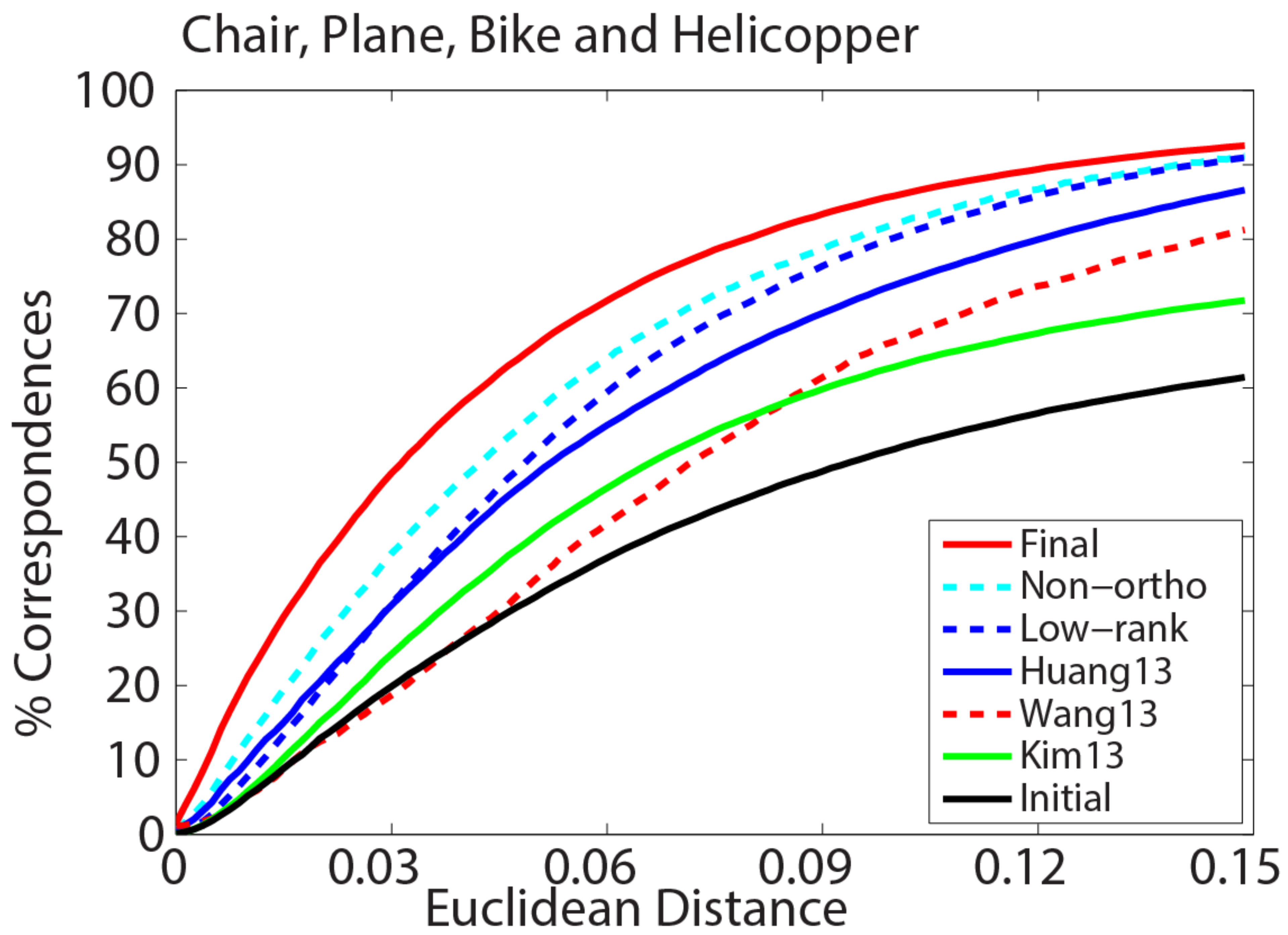
$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{11} & \cdots & \mathbf{X}_{1n} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{n1} & \cdots & \mathbf{X}_{nn} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_n \end{bmatrix} \begin{bmatrix} \mathbf{Y}_1^{+} & \cdots & \mathbf{Y}_n^{+} \end{bmatrix}$$

Low-rank factorization

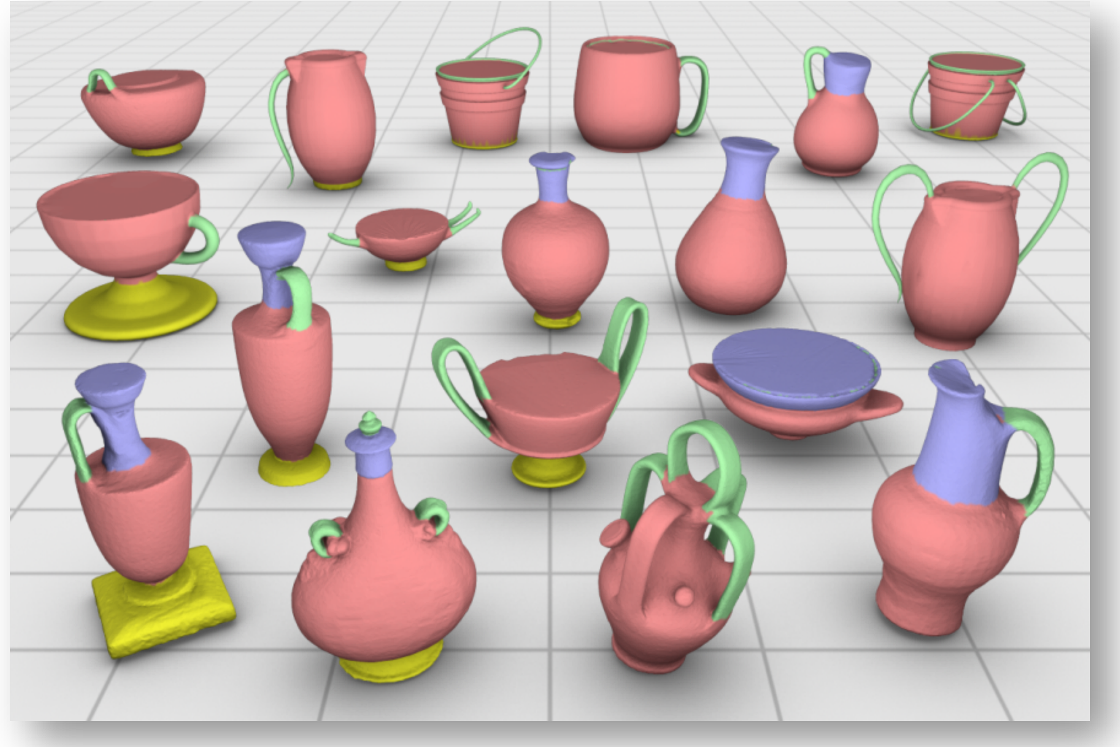


Consistent basis functions

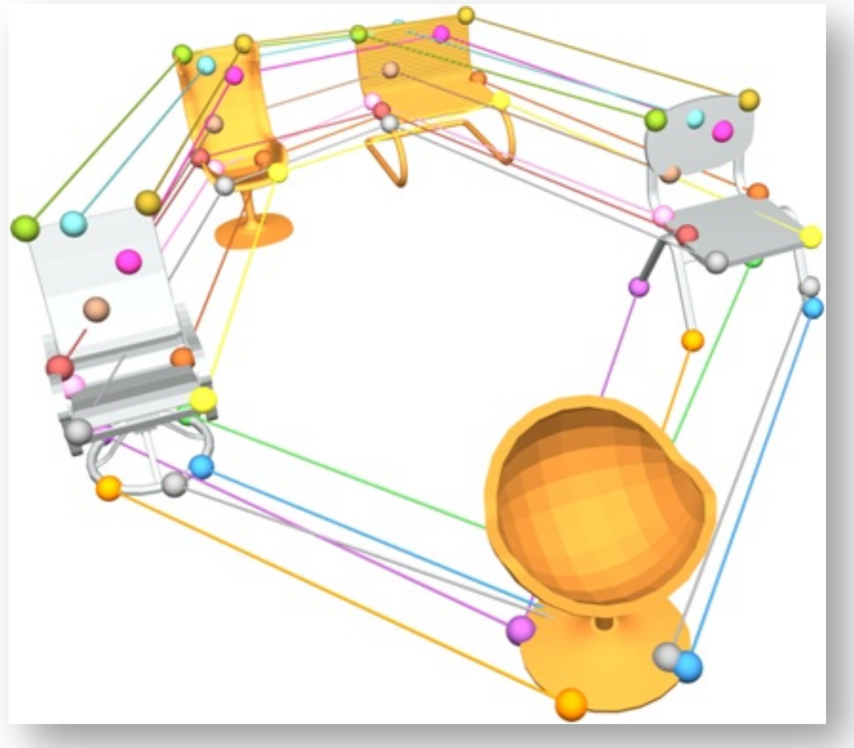
Benchmark comparison



Outline



Co-segmentation

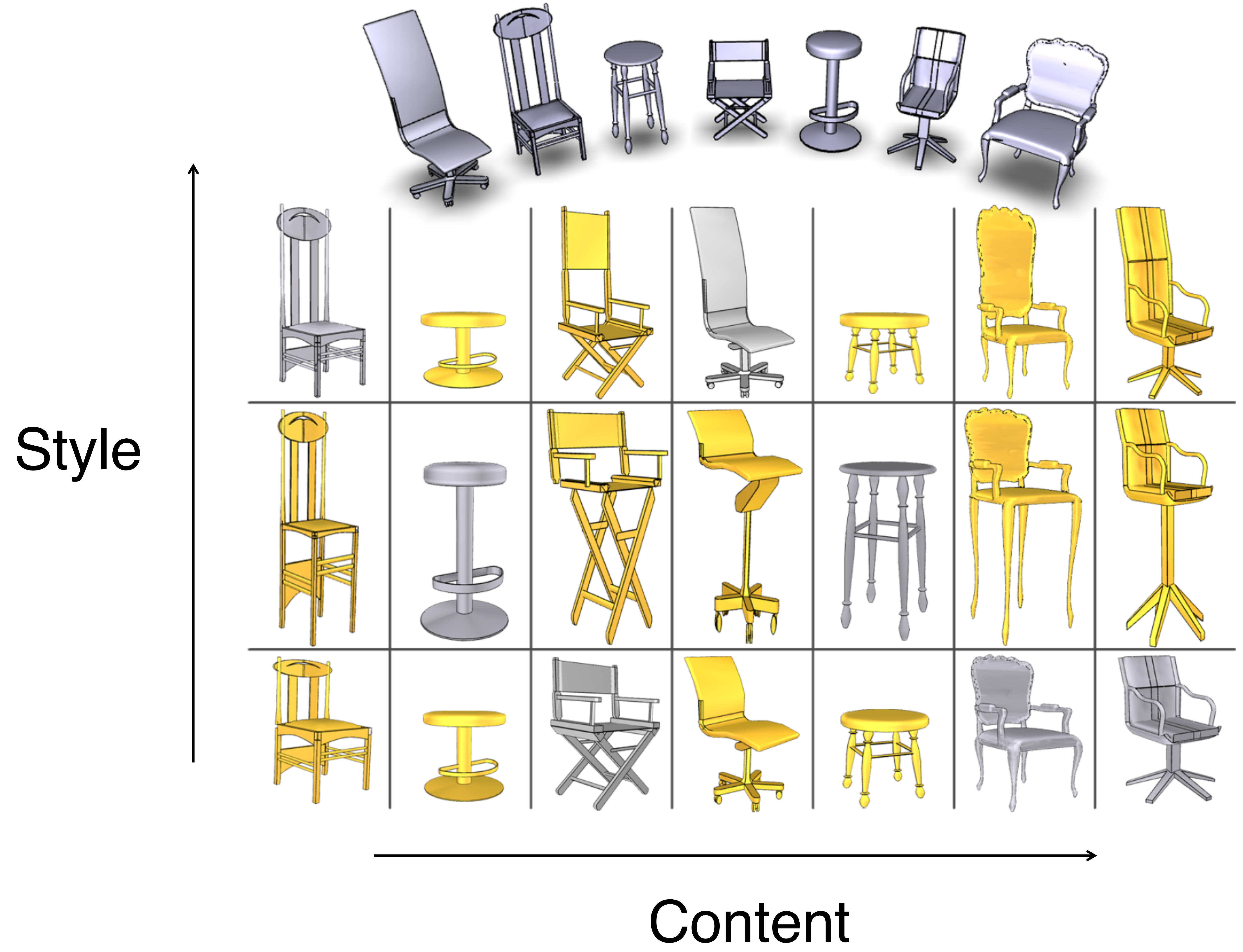


Joint matching

Other types of co-analysis

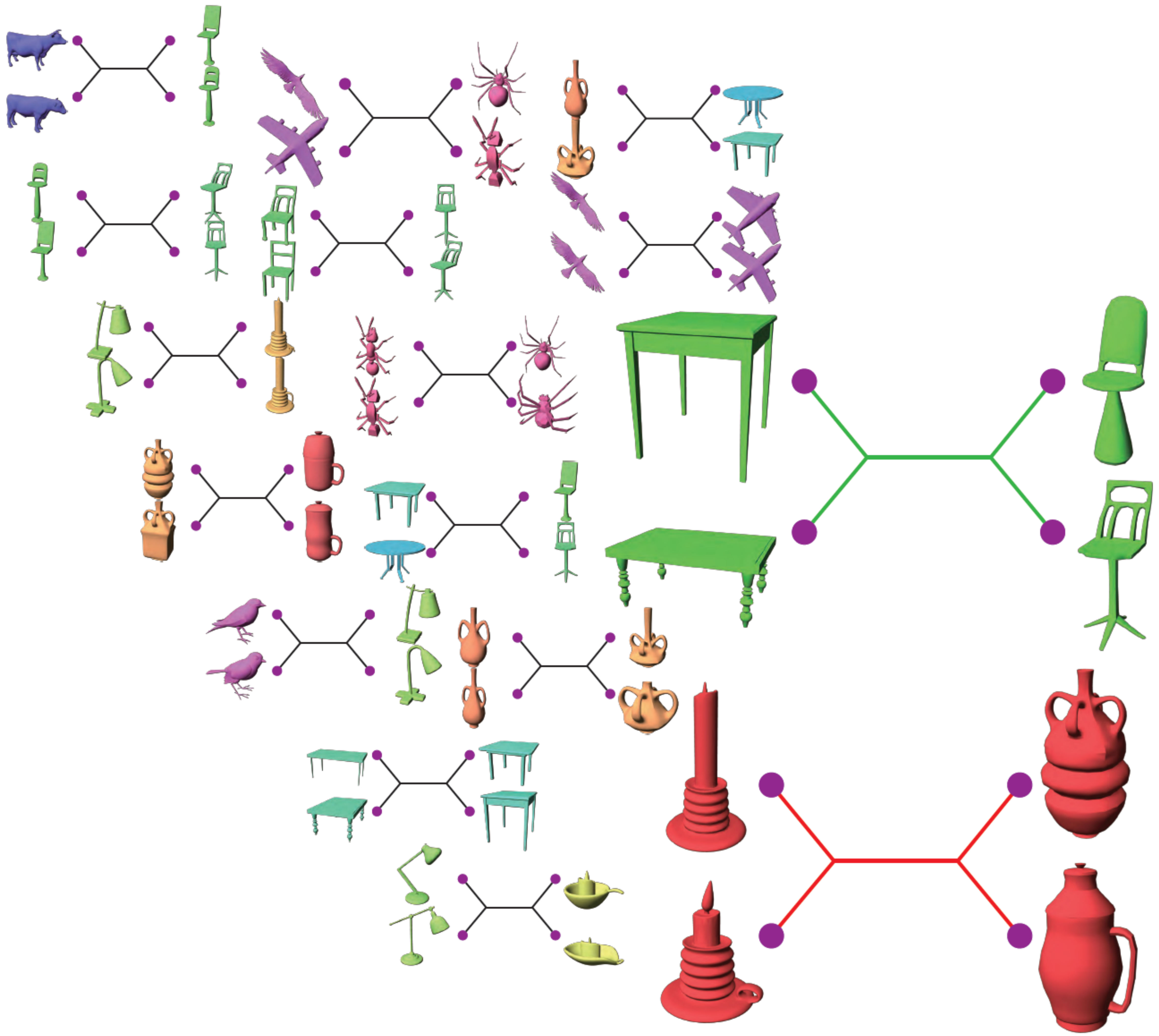
Style-content separation

[Xu et al 10]



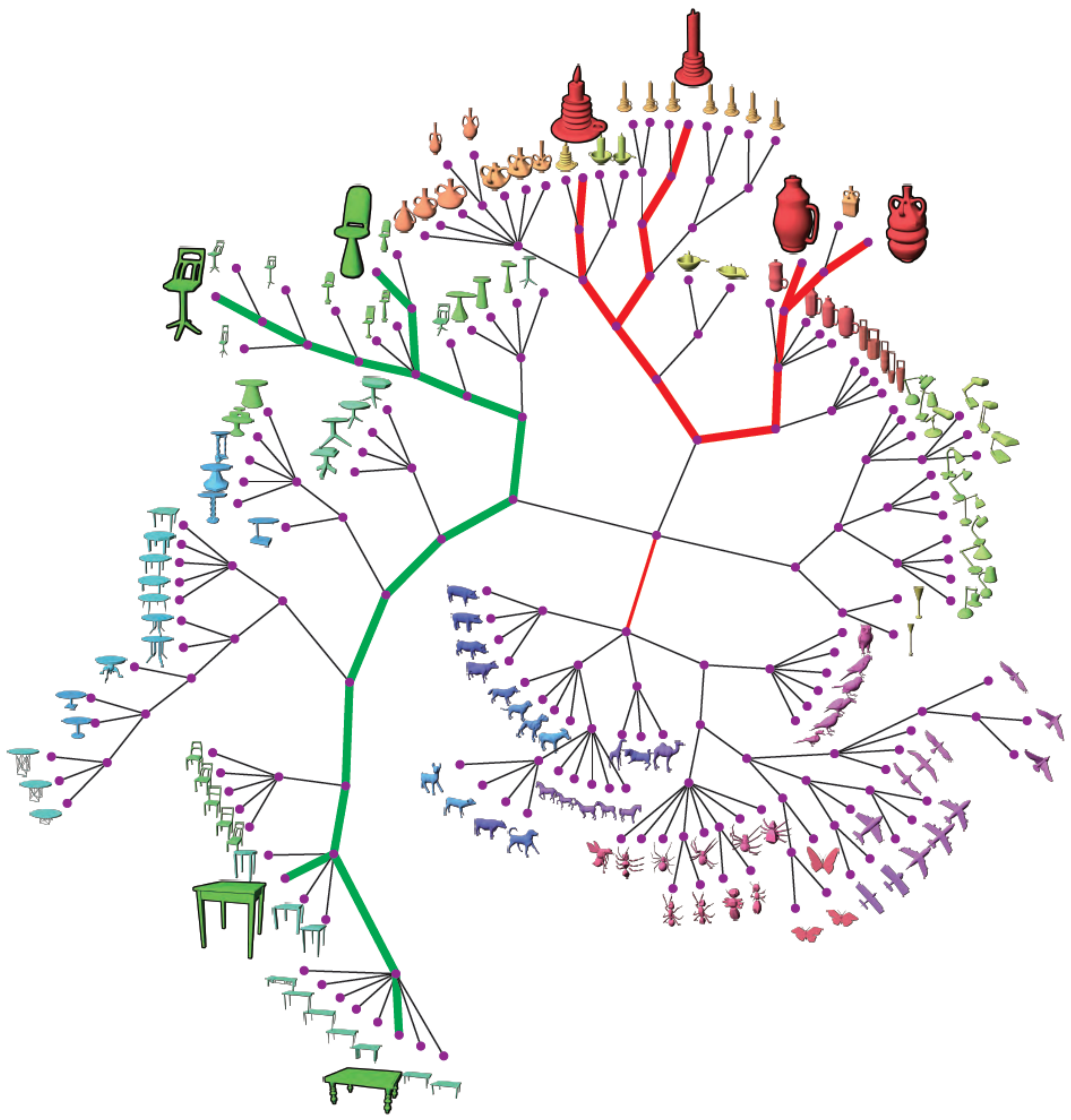
Quartet analysis

[Huang et al 13]

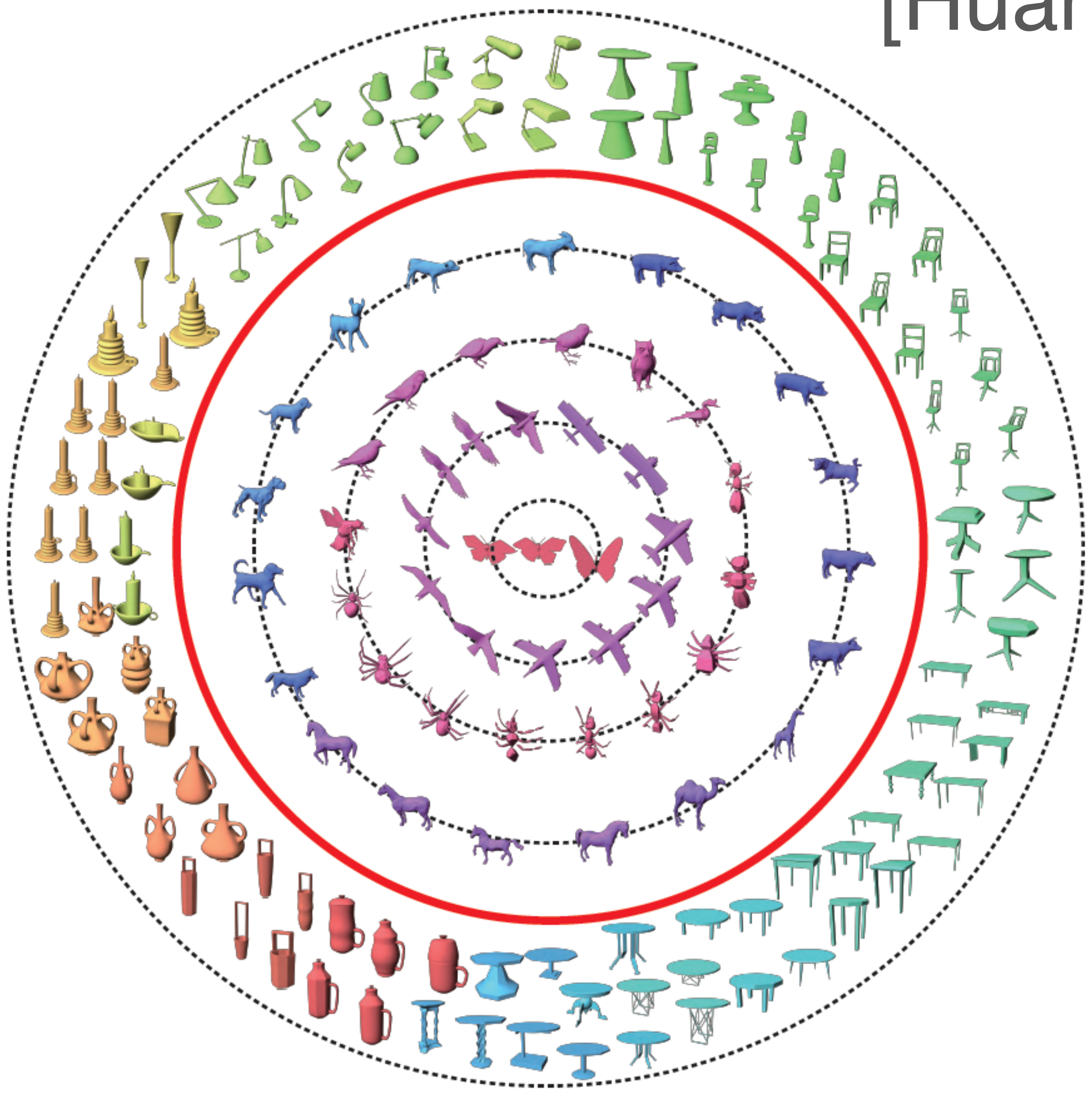


Application in organizing shapes

[Huang et al 13]

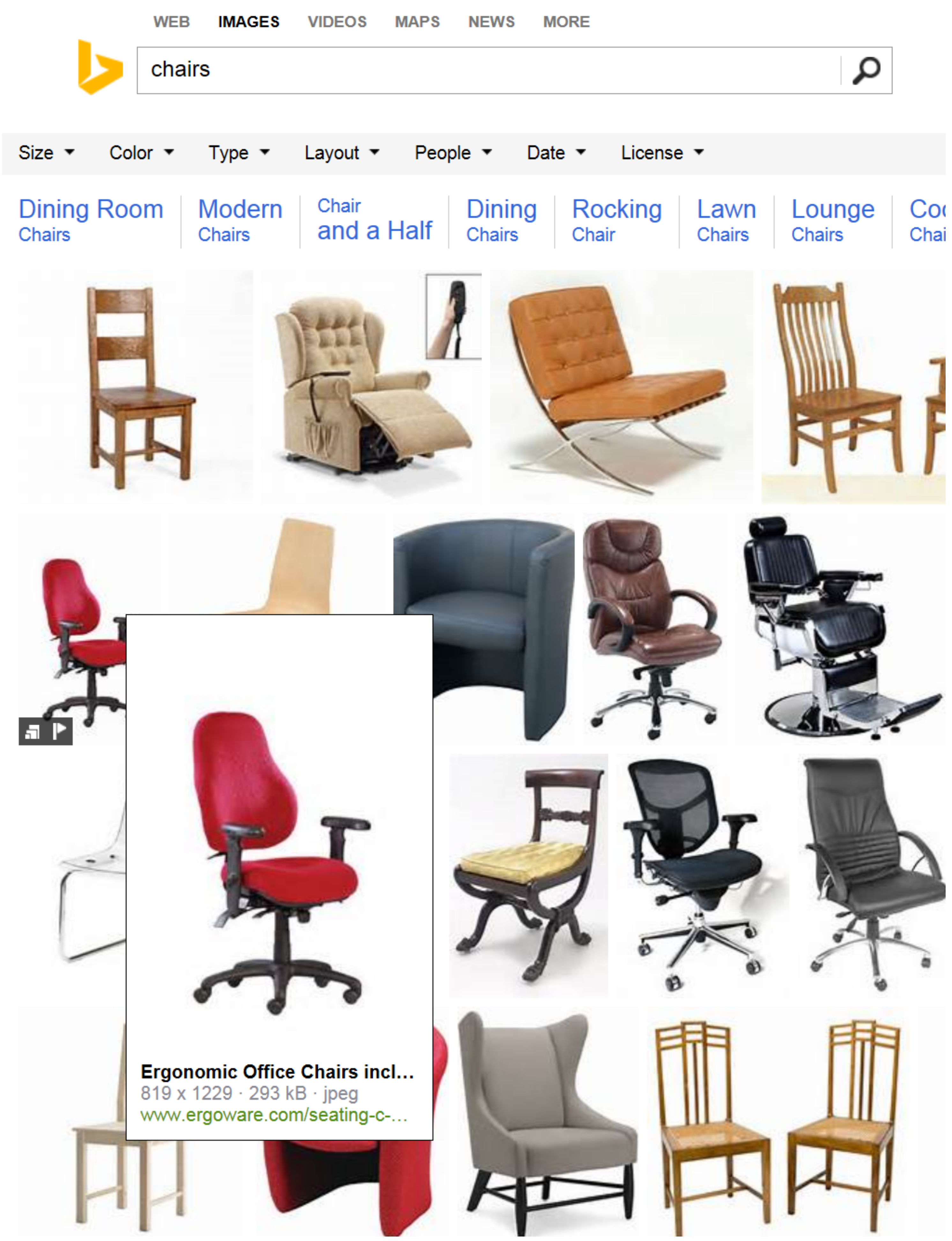


Categorization tree



Tree-distance

Co-analysis of images and shapes

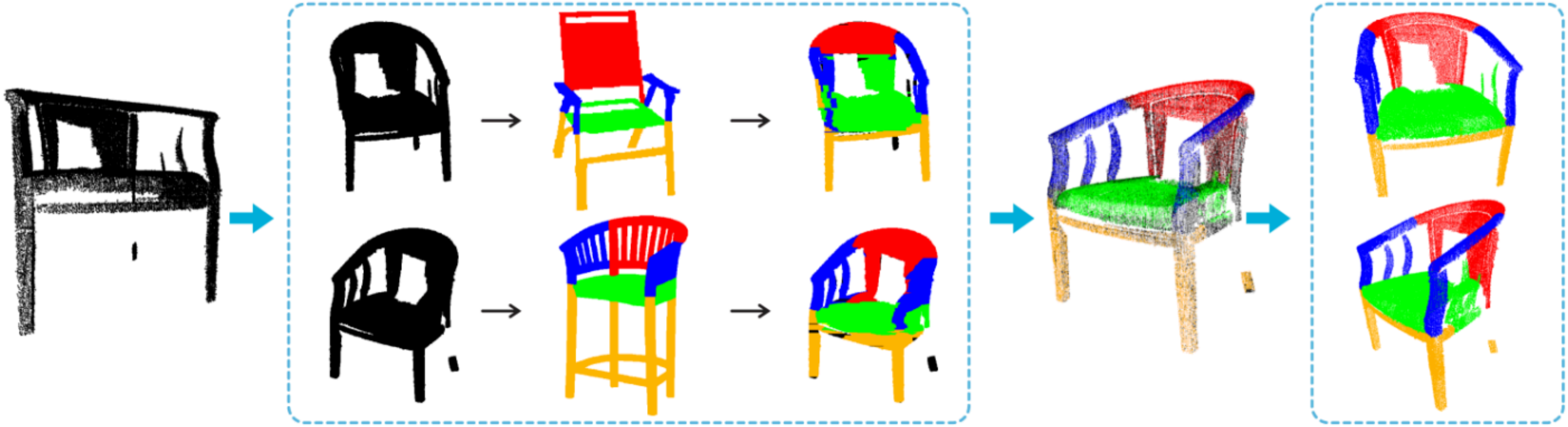


Rich textured images

Shapes

Projective shape analysis

[Wang et al. 2013]



Transferring the depth

[Su et al. 2014]